

# NUMERICAL ANALYSIS: HOMEWORK 2

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Due: February 14, 2025

## POLICIES

You may discuss the homework problems freely with other students, but please refrain from looking at their code or writeups (or sharing your own). Ultimately, you must implement your own code and write up your own solution to be turned in. Your solution, including plots and requested output from your code should be typeset and submitted via the Gradescope as a pdf file. This file must be self contained for grading. Additionally, please submit any code written for the assignment as zip file to the separate Gradescope assignment for code.

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## QUESTION 1:

When asked “is it reasonable to compute  $e^{-x}$  via its Taylor expansion” GPT-4 responds:<sup>1</sup>

Yes, it is reasonable to compute  $e^{-x}$  via its Taylor expansion. The Taylor series expansion of  $e^{-x}$  is:

$$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \dots$$

This is an infinite series that converges for all values of  $x$ , so you can use it to compute  $e^{-x}$  for any  $x$ .

However, be aware of the limitations:

1. **Precision:** The more terms you include in the series, the more accurate your result will be. However, if  $x$  is very large, you may need to sum a large number of terms to get an accurate result.
2. **Efficiency:** If you are computing  $e^{-x}$  for very large values of  $x$ , it may be more efficient to use other methods, such as computing  $e^x$  and then taking its reciprocal.

For small to moderate values of  $x$ , computing  $e^{-x}$  via its Taylor expansion can be a reasonable and accurate method.

Let’s ignore the efficiency part for a moment (as there are far better ways to compute  $e^{-x}$  from that standpoint). Assuming we take enough terms, do you agree that for “small to moderate values of  $x$ ” using the Taylor expansion provides an “accurate” result for  $e^{-x}$  on a computer (using, e.g., IEEE double precision)? Let’s say moderate here means on the order of 10 to 100—a reasonable interpretation. Justify your response.

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<sup>1</sup>See <https://chat.openai.com/share/dc160f6d-d73a-4729-94e5-ee16d409573c>, and remember that the output is random—you may get a different response if you ask the same question.

QUESTION 2:

When discussing  $LU$  decompositions in class, we considered matrices  $G^{(k)}$  (so-called Gauss Transforms) that when applied to a matrix  $A$  introduced zeros in the  $k^{\text{th}}$  column below the diagonal. Assuming all of the matrices involved in this problem are  $n \times n$ , we may write these matrices compactly as

$$G^{(k)} = I - \ell^{(k)} e_k^T,$$

where  $\ell^{(k)} = [0, \dots, 0, \ell_{k+1,k}, \dots, \ell_{n,k}]^T$  is zero in the first  $k$  entries. We will now prove two properties of these matrices that we used in class.

(a) Prove that  $(G^{(k)})^{-1} = I + \ell^{(k)} e_k^T$

(b) Prove that  $(G^{(1)})^{-1} (G^{(2)})^{-1} \dots (G^{(n-1)})^{-1} = I + \sum_{k=1}^{n-1} \ell^{(k)} e_k^T$