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Consider the fixed point iteration $x^{k+1} = G(x^k)$ where

$$G(x) = \frac{1}{4} \begin{bmatrix} x_1 - \cos(x_2) \\ x_2 - \sin(x_1) \end{bmatrix}$$

Argue that $\|G'(x)\|_\infty \leq \frac{1}{2}$ for all $x \in \mathbb{R}^2$. This implies that $\|G(x) - G(y)\|_\infty \leq \frac{1}{2}\|x - y\|_\infty$.