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cvx functions: $f(\alpha x + (1-\alpha) y) \le \alpha f(x) + (1-\alpha) f(y) \quad \alpha \in [0,1]$

cvx sets: $\forall x, y \in S, \ \alpha \in [0,1] \quad \alpha x + (1-\alpha) y \in S$

if f twice-diff, then f cvx iff $f''(x) \ge 0$

or $H(x)$ PSD $(H \succeq 0)$

$$\left.\begin{aligned} h_i(x) &\le 0 \quad i=1,\dots,k \\ Ax &= b \end{aligned}\right\}$$ feasible pts form a cvx set

$$\begin{aligned} \min \ & f(x) \\ \text{s.t. } & x \in S \end{aligned}$$ $\Rightarrow$ local min are global min!

finding feasible point: $S = \{ A \mid x^T A x \ge 0 \text{ if } x \ge 0 \}$

$$\underbrace{x^T \alpha A x}_{\ge 0} + \underbrace{x^T (1-\alpha) B x}_{\ge 0}$$

$A \in S$? (co-)NP-hard

Logistic regression

Data $(x_1, y_1), \ldots, (x_n, y_n)$ $\quad x_i \in \mathbb{R}^d, \quad y_i \in \{0,1\}$

$\Pr(Y=1 \mid X=x) \equiv p(x)$

① $p(x) = \cancel{x^T\beta}$ ② $\log\left(\frac{p(x)}{1-p(x)}\right) = x^T\beta$

$$\frac{p(x)}{1-p(x)} = e^{x^T\beta} \Rightarrow \boxed{p(x) = \frac{1}{1+e^{-x^T\beta}}} \quad 1-p(x) = \frac{1}{1+e^{x^T\beta}}$$

$$L(\beta; \{(x_i, y_i)\}) = \prod_{i=1}^{n} p(\beta; x_i)^{y_i} (1-p(\beta; x_i))^{1-y_i}$$

$$\Rightarrow N(\beta) = -\left[\sum_{i=1}^{n} y_i \log\left(\frac{1}{1+e^{-x^T\beta}}\right) + (1-y_i)\log\left(\frac{1}{1+e^{x^T\beta}}\right)\right]$$

$$= \sum_{i=1}^{n} \log(1+e^{x^T\beta}) - \sum_{i=1}^{n} y_i \Big[\underbrace{\log(1+e^{-x^T\beta}) + \log(1+e^{x^T\beta})}\Big]$$

$$\frac{e^z}{1+e^z} =$$

$$\min \|Ax-b\|_2^2$$
$$\text{s.t.} \quad \|x\|_2 \leq t$$

nonzero