May 3, 2021

$$\min f(x)$$
$$\text{s.t.} \quad c_i(x) = 0 \quad i = 1, \dots, m$$
$$h_i(x) \le 0 \quad i = 1, \dots, k$$

$\Downarrow$

gradient, Newton, SQP, KKT, barrier

convex optimization

$$\min c^T x$$
$$\text{s.t.} \quad Ax \le b$$

$\Downarrow$

simplex, ellipsoid

$$\min \|Ax - b\|_2$$

$\Downarrow$

QR

$$\min f(x)$$
$$\text{s.t.} \quad c_i(x) = 0$$
$$h_i(x) \le 0$$

f, c_i, h_i are convex functions

structure for algorithms

broad enough model

Convex functions

f concave if $-f$ is convex

$f: \mathbb{R}^n \to \mathbb{R}$ is convex if for any $\alpha \in [0,1]$

$$f(\alpha x + (1-\alpha)y) \leq \alpha f(x) + (1-\alpha) f(y)$$

LPs are $\downarrow$ convex opt

Example: $f(x) = a^T x + b$

$$f(\alpha x + (1-\alpha)y) = a^T(\alpha x + (1-\alpha)y) + b \quad \alpha f(x) \quad (1-\alpha)f(y)$$

$$= \alpha a^T x + (1-\alpha) a^T y + \alpha b + (1-\alpha) b$$

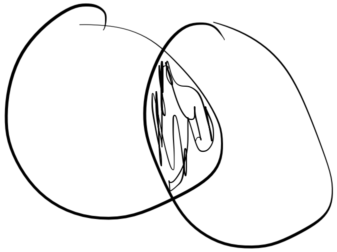
Claim: f twice diff, f convex if and only if H is SPsD

Proof: Taylor's theorem

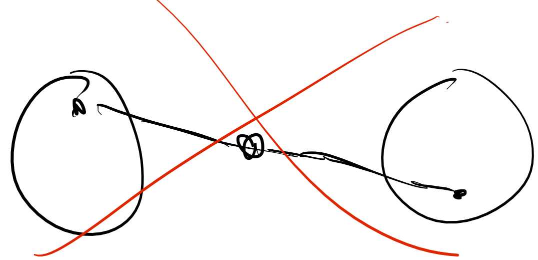
Example: $f(x) = x^T A x + x^T b$

, $\max(f, g)$ convex

$$\max(f(\alpha x + (1-\alpha)y), g(\alpha x + (1-\alpha)y))$$



Union



$$h(x), h(y) \le 0 \qquad h(\alpha x + (1-\alpha) y)$$
$$\le \alpha h(x) + (1-\alpha) h(y)$$
$$\le 0$$

$$\{x \mid h_i(x) \le 0,\ i=1,\ldots,k\}$$