April 30, 2021

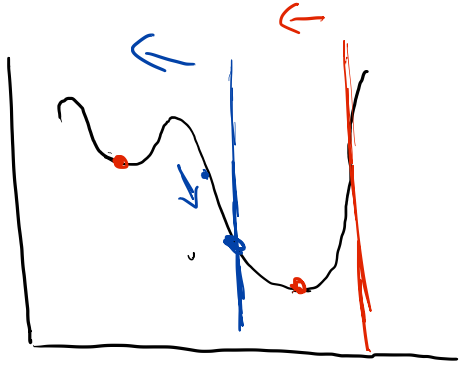
$$\min f(x)$$
$$\text{s.t. } h_i(x) \le 0 \quad i = 1, \dots, k$$

$$\min f(x)$$
$$\text{s.t. } h(x) \le 0$$

At x^*:

① $h(x^*) < 0$ (inactive)

② $h(x^*) = 0$ (active)



If inactive, can't have $\nabla f(x^*)^T p < 0$

active can't have p s.t.

(i) $\nabla f(x^*)^T p < 0$

(ii) $\nabla h(x^*)^T p < 0$

$$\nabla f = -\mu \nabla h$$
$$\mu \ge 0$$

$$L(x, \mu) = f(x) + \sum_{i=1}^{k} \mu_i h_i(x)$$

$$\nabla_x L(x^*, \mu^*) = 0$$
$$h_i(x^*) \le 0$$
$$\mu_i^* \ge 0$$
$$h_i(x^*)\, \mu_i^* = 0$$

Idea: guess active constraints (at opt)
treat active as equality
ignore

Problem: 2^k possible active constraint sets

$$\min f(x)$$

$$\text{s.t. } c_i(x) = 0 \quad i = 1, \dots, m$$

$$h_i(x) \le 0 \quad i = 1, \dots, k$$

$$\min f(x) - \gamma \sum \log(-h_i(x)) + \mu \sum c_i^2(x) \quad \gamma \to 0 \quad \mu \to \infty$$