April 28, 2021

$$\min \; f(x)$$
$$\text{s.t.} \quad c_i(x) = 0 \quad i = 1, 2, \dots, m$$
$$h_i(x) \le 0 \quad i = 1, 2, \dots, k$$

Last time: necessary conditions for local min

Lagrangian:

$$L(x, \lambda, \mu) = f(x) + \sum_{i=1}^{m} \lambda_i c_i(x) + \sum_{i=1}^{k} \mu_i h_i(x)$$

KKT conditions:

$$\nabla_x L(x^*, \lambda^*, \mu^*) = 0$$
$$c_i(x^*) = 0$$
$$h_i(x^*) \le 0$$
$$\mu_i^* \ge 0$$
$$h_i(x^*) \mu_i^* = 0$$

$$\min_{w, v} \; w^T A v \qquad A \text{ full rank}$$
$$\text{s.t.} \quad w^T w = 1 \quad v^T v \ge 1$$

$$L(x, \lambda, \mu) = w^T A v + \lambda (w^T w - 1) + \mu (1 - v^T v)$$

$$\nabla_w L = Av + 2\lambda w = 0$$
$$\nabla_v L = A^T w - 2\mu v = 0$$

$$A^T A v = -2\lambda A^T w = (-4\lambda\mu) v$$
$$A A^T w = 2\mu A v = (-4\lambda\mu) w$$

$$A = U \Sigma V^T \quad A^T A = V \Sigma^2 V^T \quad A A^T = U \Sigma^2 U^T$$

$$w = u_j \quad \lambda = -\tfrac{1}{4}\sigma_j \quad w^T w - 1 = 0$$
$$v = v_j \quad \mu = \sigma_j \quad 1 - v^T v = 0$$

choose $w = u_n \quad v = v_n$

$$w^T A v = \sigma_n$$

$A^T\lambda$

$Hx + g$

$$\begin{pmatrix} H & A^T \\ A & 0 \end{pmatrix}\begin{pmatrix} x \\ \lambda \end{pmatrix} = \begin{pmatrix} -g \\ b \end{pmatrix}$$

$= Q_1 R_1 v = b$

$v = R_1^{-1} Q_1^T b$

all feasible points:

$$x = \begin{pmatrix} v \\ 0 \end{pmatrix} + Q_2 z = y + Q_2 z$$

$$\min_z \frac{1}{2}(y + Q_2 z)^T H (y + Q_2 z) + g^T (y + Q_2 z) + c$$

$\vdots$

$$\frac{1}{2} z^T Q_2^T H Q_2 z + (g + Q_2^T H y)^T z$$

$$\min_z f(y + Q_2 z)$$

$$Q_2^T H Q_2 z = -(Q_2^T H y + g)$$