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So far: $\min_{x \in \mathbb{R}^n} f(x)$ $\quad f: \mathbb{R}^n \to \mathbb{R}$

Next: $\min_{x \in \mathbb{R}^n} f(x)$ objective (function)

s.t. $c_i(x) = 0$ equality constraints

$c_i(x) \geq 0$ inequality "

Special cases: c_i linear $x^T v = 0 \quad \geq 0$

convex

General strategies (before algos)

① Transforming / eliminating variables

$$\alpha > 0 \Rightarrow \exp(u) \qquad \alpha = \log(u)$$

$$|x| \le 1 \Rightarrow x \le 1 \qquad -1 \le x$$

$$-x + 1 \ge 0 \qquad x + 1 \ge 0$$

② Move constraints into objective

log barrier

$$\min f(x) \quad \text{s.t. } c(x) \le 0$$

$$\min_x f(x) + \mu \log(-c(x))$$

$$\mu > 0 \quad c(x) \to 0^- \quad \log(-c(x)) \to \infty$$

$$\mu \to 0$$

$$\min_x f(x) + \frac{1}{\mu} \max(c(x), 0)$$

$$\mu \to 0$$

③ Add variables (Lagrange multipliers)

$$\min f(x) \qquad x_1, x_2, \ldots$$
$$\text{s.t. } c(x) \le 0$$

$$c(x_*) < 0 \qquad \nabla f(x_*) = 0$$

active

$c(x_*) = 0$ Can we move off 0 and improve objective?

①

$$f(x_* + p) \approx (x_*) + \nabla f(x_*)^T p < 0$$ ②