April 16, 2021

$K_k(A,b) = \text{span}\{b, Ab, \dots, A^{k-1}b\}$

Q_k basis

$AQ_k = Q_{k+1}\bar{H}_k \quad \bar{H}_k = \begin{bmatrix} H_k \\ h_{k+1,k}e_k^T \end{bmatrix}$

$Q_k^T A Q_k = H_k$

$A = A^T \Rightarrow H_k = T_k =$ (≡)

$AQ_k e_k = Q_{k+1}\bar{H}_k e_k$

$Aq_k = h_{k+1,k}q_{k+1} + h_{k,k}q_k + h_{k-1}q_{k-1}$

$x_k = \underset{x}{\text{argmin}} \ \frac{1}{2}x^TAx - x^Tb$
s.t. $x \in K_k$

$x = Q_k c$

$\min \frac{1}{2}c^T \underbrace{Q_k^T A Q_k}_{T_k} c - c^T Q_k^T b$

$T_k c = \beta_0 e_1$

(≡)

$q_1 = \frac{b}{\|b\|_2}$

$*, x_2, \dots, x_k, \dots$

Conjugate gradient method (CG)

$x_0 = 0,\ r_0 = b \quad p_0 = r_0$

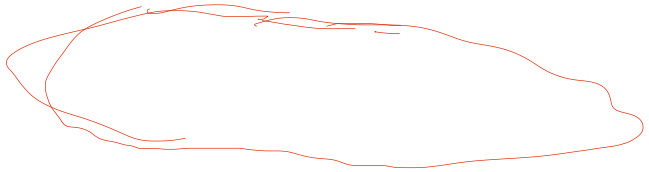
for $k = 1, 2, \ldots$

- $\alpha_k = r_{k-1}^T r_{k-1} / p_{k-1}^T A p_{k-1}$
- $x_k = x_{k-1} + \alpha_k p_{k-1}$
- $r_k = r_{k-1} - \alpha_k A p_{k-1}$ ($Ax_k - b$)
- $\beta_k = r_k^T r_k / r_{k-1}^T r_{k-1}$
- $p_k = r_k + \beta_k p_{k-1}$

(1) converge in n steps

(2) $p_k^T A p_j = 0 \quad j < k$

(A-conjugacy)



$k \times k$

$x = Q_k c$

approx evec

λ approx eval