

April 14, 2021

$$Ax = b$$

$$A = M - N \quad Mx_{k+1} = Nx_k + b \quad x_{k+1} = \overbrace{M^{-1}N}^R x_k + M^{-1}b$$

$$e_{k+1} = Re_k = R^k e_0 \quad (e_k = x_k - x_*)$$

$$e_{k+1} \approx p(R) e_k \quad p(R) = \max_j |\lambda_j(R)| \quad (< 1 \text{ for convergence})$$

$$x_k - x_{k+1} = e_k - e_{k+1} \approx (1 - p(R)) e_k$$

$$x_* = x_k - e_k \approx x_k = \frac{(x_k - x_{k-1})}{1 - p(R)}$$

Can we do better by combining iterates?

If  $e_k \rightarrow 0$ , can't really do worse

$$x_{k+1} \rightarrow x_k \xrightarrow{R x_k + M^{-1}b} x_{k+1}$$

Recall heavy ball:  $x_{k+1} = \underline{x_k} = \alpha_k \nabla f_k + \beta_k (x_k - x_{k-1})$

$$K_k = \text{span} \{x_0, \dots, x_k\}$$

(subspace)

$$x_{k+1} = \underset{x}{\text{argmin}} f(x) = \frac{1}{2} x^T A x - x^T b$$

s.t.  $x \in K_k$

$$x_{k+1} = \sum_{j=0}^{k+1} c_j R^j M^{-1} b$$

① How good is this? \*

② How to solve?

③ Efficiency? (growing  $k$ )

$$K_k(A, b) = \text{span} \{b, Ab, \dots, A^{k-1}b\}$$

$K_{k+2}(\dots)$

$$x_{k+1} = R x_k + M^{-1} b$$

$$x_0 = M^{-1} b \Rightarrow x_1 = R x_0 + M^{-1} b = R M^{-1} b + M^{-1} b = (R + I) M^{-1} b$$

$$x_2 = R x_1 + M^{-1} b$$

$$= (R^2 + R + I) M^{-1} b$$

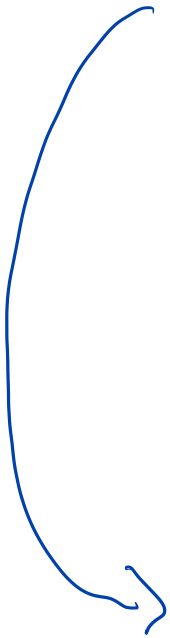




[

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(  $e_k$  (  $\circ$  )  $e_k$



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