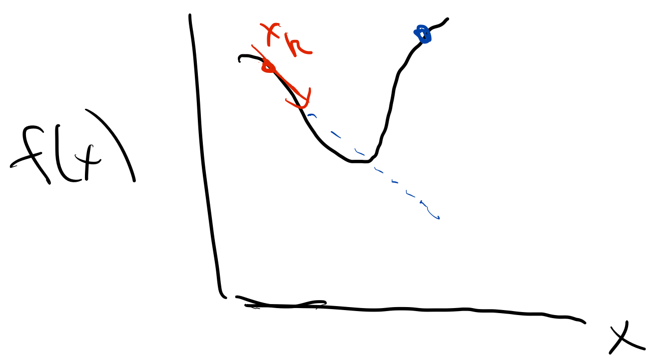
April 9, 2021

$$\min_{x \in \mathbb{R}^n} f(x) \qquad \nabla f(x) = 0$$

$$x_{k+1} = x_k + \alpha_k p_k \qquad p_k \to \begin{cases} -H_k^{-1} g_k \\ -\tilde{H}_k^{-1} g_k \\ -B_k^{-1} g_k \\ -g_k \end{cases}$$

Exact line search $\alpha_k = \arg\min_{\alpha} f(x_k + \alpha p_k)$

Inexact " "

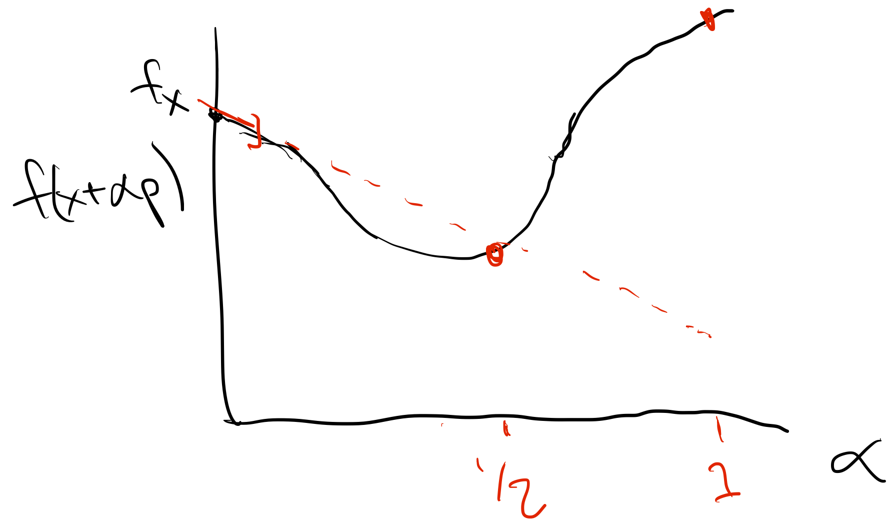


① Not too short (little progress)

② Not too long

Backtracking

```
find_step_size(f, x, p)
    f_x = f(x)
    α = 1.0
    while f(x + αp) > f_x
        α = α / 2
    return α
```



for $k = 1, 2, \ldots$ Examples:

Get p_k $(-g_k, -H^{-1} g_k)$

$\alpha_k = \text{find\_step\_size}(f, x_k, p_k)$

$x_{k+1} = x_k + \alpha_k p_k$

Armijo

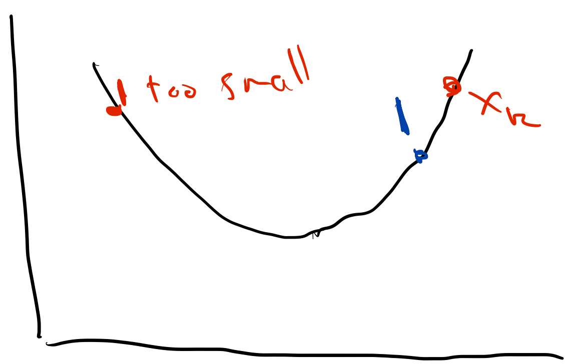
Idea: a "sufficient decrease"

Require:

$$f(x_k + \alpha_k p_k) \le f(x_k) + c_1 \underbrace{\alpha_k}_{>0} \underbrace{g_k^T p_k}_{<0}$$

$c_1 \in (0,1)$ fixed

(often small, 1e-3, 1e-4)



$$\phi(\alpha) = f(x_k + \alpha p_k)$$
$$\phi'(\alpha) = \nabla f(x_k + \alpha p_k)^T p_k$$
$$\phi'(0) = g_k^T p_k$$

Curvature

Don't want steps too small

$$\phi'(\alpha_k) \geq c_2 \phi'(0)$$

$c_2 \in (c_1, 1)$ fixed

$\phi'(\alpha)$