March 29, 2021 $\quad g(x) = 0 \quad (\text{Ex: } g = f')$

- Taylor: $\underset{0}{\underset{\shortparallel}{g(x+h)}} = g(x) + g'(x)h + O(\cancel{h}^2)$

$$\Rightarrow \quad h = -g(x)/g'(x)$$

$$x_k = x \qquad x_{k+1} = x_k + h$$

$$\Rightarrow \quad x_{k+1} = x_k - g(x_k)/g'(x_k) \qquad \text{Newton's method (root-finding)}$$

$$g = f' \qquad x_{k+1} = x_k - f'(x_k)/f''(x_k) \qquad \text{"} \qquad \text{"} \qquad (\text{for 1-D opt})$$

(1) $\quad 0 = g(x_*) = g(x_k) + g'(x_k)(x_* - x_k) + \frac{1}{2} g''(z) (\underbrace{x_* - x_k}_{e_k})^2$

(2) $\quad 0 \approx g(x_k) + g'(x_k)(\overbrace{x_{k+1} - x_k}^{h})$

(2)−(1)

$$0 = g'(x_k)(\underbrace{-x_* + \cancel{x_k} + x_{k+1} - \cancel{x_k}}_{-e_{k+1}}) - \frac{1}{2} g''(z)(x_* - x_k)^2$$

$$|e_{k+1}| = \left|\tfrac{1}{2} g''(z)\right| e_k^2 / |g'(x_k)|$$

Newton: $x_{k+1} = x_k - g(x_k)/g'(x_k)$

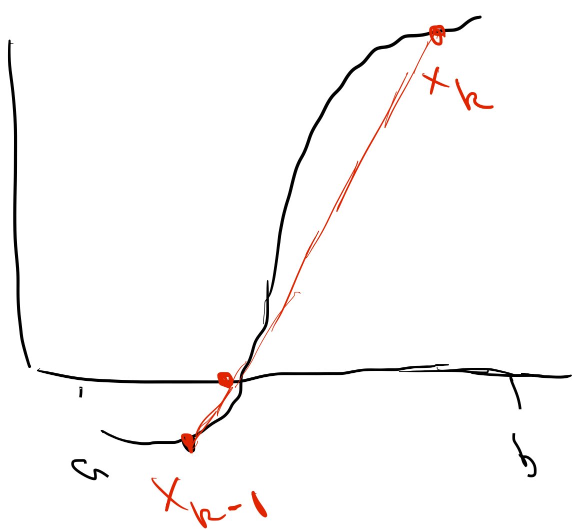
What if no derivative access?

$$g'(x) = \lim_{\varepsilon \to 0} \frac{g(x+\varepsilon) - g(x)}{\varepsilon} \qquad g'(x) \approx \frac{g(x+\varepsilon) - g(x)}{\varepsilon}$$

$$x_{k+1} \approx x_k - \frac{g(x_k)}{[g(x_k+\varepsilon) - g(x_k)]/\varepsilon}$$

$g(x_0), g(x_1), \ldots, g(x_{k-1}), g(x_k)$

$$\varepsilon = x_{k-1} - x_k \Rightarrow x_{k+1} = x_k - \boxed{\frac{g(x_k)(x_{k-1} - x_k)}{g(x_{k-1}) - g(x_k)}}$$



outside $[a, b]$

MVT:

$x_1 x_2$