March 26, 2021

Norms: $\|A\|_2 = \max_{x} \|Ax\|_2 = \sigma_1(A)$ $\quad x = v_1$

s.t. $\|x\|_2 = 1$

Least squares: $\arg\min_x \|Ax - b\|_2^2 \; (+\beta \|x\|_2^2)$ $\quad \hat{x} = R^{-1}Q^T b$

$\begin{pmatrix} A \\ \sqrt{\beta} I \end{pmatrix} = QR$

Eigenvalues / eigenvectors $\quad \min_x x^T A x$ $\quad \lambda_n \quad Ax = \lambda_n x$

s.t. $\|x\|_2 = 1$

optimization variable $\rightarrow$ $\arg\min_{x} \; x^T L x$ objective function

s.t. $\|x\|_2^2 = n$ constraints

linear const. $\; e^T x = 0$

unconstrained

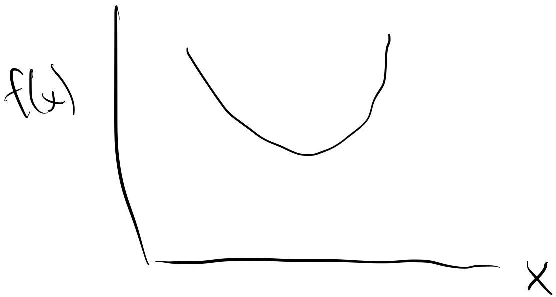
$\min_x f(x)$

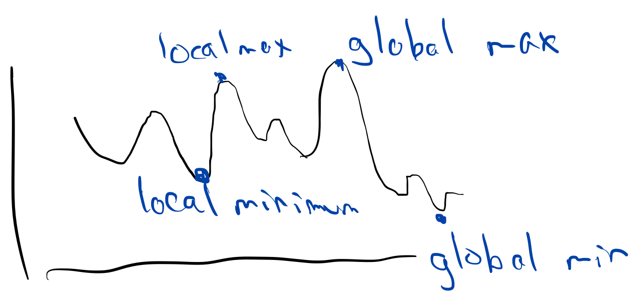
$\Longleftrightarrow \max_x -f(x)$

constrained

$\min f(x)$

s.t. $g(x) = 0$





$f: \mathbb{R} \to \mathbb{R}$ $\qquad \min_{x \in \mathbb{R}} f(x)$

Taylor expansion: $f(x) = \sum_{j=0}^{\infty} \frac{f^{(j)}(a)}{j!} (x-a)^j$

Example: e^{-x} $(a=0)$: $1 - x + \frac{x^2}{2} - \frac{x^3}{6} + \dots$

$$f(x+h) = \sum_{j=0}^{\infty} \frac{f^{(j)}(x)}{j!} h^j \approx f(x) + \underline{f'(x)}h + \frac{1}{2} f''(x) h^2$$

$$= (b_0 - a_0)/2^k$$