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$$T_0 = A \in \mathbb{R}^{n\times n}$$

for $k = 0, 1, 2, \dots$

$$Q_{k+1} R_{k+1} = T_k - \mu_k I$$

$$T_{k+1} = R_{k+1} Q_{k+1} + \mu_k I$$

① $O(n^3)$ per iteration

② Complex eigenvalues

$B = SAS^{-1}$ precompute

$$Q^T A Q =$$

$$\begin{pmatrix} x & x & x & x \\ x & x & x & x \\ x & x & x & x \\ x & x & x & x \end{pmatrix} \xrightarrow{Q_1^T A} \begin{pmatrix} x & x & x & x \\ \cancel{x} & x & x & x \\ \cancel{x} & x & x & x \\ \cancel{x} & x & x & x \end{pmatrix} \leftarrow \underbrace{Q_1^T A Q_1 e_1}$$

$$Q^T = I - 2vv^T / v^T v \qquad v = \|a_1\|_2 e_1 - a_1$$

$$A = \begin{pmatrix} x & x & x & x \\ x & x & x & x \\ x & x & x & x \\ x & x & x & x \end{pmatrix} \xrightarrow{\begin{pmatrix} 1 & 0 \\ 0 & Q_1^T \end{pmatrix} A} \begin{pmatrix} x & x & x & x \\ x & x & x & x \\ 0 & x & x & x \\ 0 & x & x & x \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & Q_1 \end{pmatrix}$$

$$\begin{pmatrix} x & x & x & x \\ x & x & x & x \\ 0 & x & x & x \\ 0 & x & x & x \end{pmatrix} \xrightarrow{\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ & & Q_2^T \end{pmatrix}} \begin{pmatrix} x & x & x & x \\ x & x & x & x \\ 0 & x & x & x \\ 0 & 0 & x & x \end{pmatrix}$$

upper Hessenberg form

$$Q^T A Q = H = \qquad i > j+1 \Rightarrow H_{ij} = 0$$

$O(n^3)$ pre-compute $T_0 = H$

$$A = A^T$$

$$(Q^T A Q)^T = H^T =$$

$$\|$$

$$Q^T A^T Q = Q^T A Q = H$$

tridiagonal form

$$Q^T A Q = T =$$

$$\begin{pmatrix} c & -s \\ s & c \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \to \begin{pmatrix} \|x\|_2 \\ 0 \end{pmatrix}$$

k

$$c^2 + s^2 = 1$$

$$\begin{array}{c} \\ i \\ \\ j \\ \end{array}\begin{array}{c} \begin{array}{ccccccc} & & i & & j & & \end{array} \\ \begin{pmatrix} 1 & & & & & & \\ & \ddots & & & & & \\ & & c & & -s & & \\ & & & 1 & & & \\ & & & & \ddots & & \\ & & s & & c & & \\ & & & & & \ddots & \\ & & & & & & 1 \end{pmatrix} \end{array} = G^T$$

$G^T A$ leave row k same, $k \neq i, j$

AG " col " " "

$$\begin{pmatrix} x & x \\ x & x \\ 0 & x \\ 0 & \cdot \end{pmatrix}$$

$$\xrightarrow{G_1^T H} \begin{pmatrix} x & x & x & x \\ 0 & x & x & x \\ 0 & x & x & x \\ 0 & 0 & x & x \end{pmatrix}$$

$$\xrightarrow{G_2^T G_1^T H} \begin{pmatrix} x & x & x & x \\ 0 & x & x & x \\ 0 & 0 & x & x \\ 0 & 0 & x & x \end{pmatrix}$$

$$G_{n-1}^T \cdots G_1^T H = R \quad O(n^2)$$