March 22, 2021

- **Subspace iteration**

$$\left\{ Q_0 \in \mathbb{R}^{n \times d} \quad Q_0^T Q_0 = I \right.$$

for $k = 0, 1, 2, \ldots$

$$Z_k = A Q_k$$

$$Q_{k+1} R_{k+1} = Z_k$$

$$\left| \lambda_d / \lambda_{d+1} \right|^k \to 0$$

$d = 1 \Rightarrow$ power method

$d = n$

$$\underset{n}{\overset{n}{\boxed{A}}} \; \underset{n}{\overset{n}{\boxed{Q_k}}} = \underset{n}{\overset{n}{\boxed{Q_{k+1}}}} \; \underset{n}{\overset{n}{\boxed{R_{k+1}}}}$$

$$A = Q_{k+1} R_{k+1} Q_k^T$$

$$Q_k = Q_{k+1} = Q \qquad A = Q \boxed{\diagdown} \, Q^T$$

Schur form!

① $Q_k^T A Q_k = T_k \to T$

② Diagonal of $T \Rightarrow$ evals of $A$

③ $T_k \Rightarrow$ approx evals of $A$

$$T_{k+1} = Q_{k+1}^T A Q_{k+1}$$

$$= \underbrace{Q_{k+1}^T A Q_k}_{R_{k+1}} \underbrace{Q_k^T Q_{k+1}}_{Q_{k+1}}$$

- **QR iteration**

$$T_0 = A$$

for $k = 0, 1, 2, \ldots$

(QR) $Q_{k+1} R_{k+1} = T_k$ $\quad R_{k+1} = Q_{k+1}^T T_k$

(Matmul) $T_{k+1} = R_{k+1} Q_{k+1}$

$$= Q_{k+1}^T T_k Q_{k+1}$$

Can show: $Q_{k+1} = Q_1 \cdots Q_k$

$R$ same as in SF $(d = n)$

Things to worry about

① QR fact + matmul each step $\Rightarrow O(n^3)$ per step

$Q_0^T \; Q_0$

$T_R V$ $\quad Q_0$ $\quad (Q_0$

③ Stable?

just QR, mult by orthog matrices

$$(M = Q B Q^T) \quad B \text{ symm}$$

$$M^T = (Q^T)^T B^T Q^T$$

$$= Q B Q^T$$