Feb 24, 2021 $\quad Ax = b \quad LUx = b$

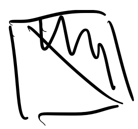
$L = I$

for $j = 1:(n-1)$

$\quad L(j+1:n,\ j) = A(j+1:n)\,/\,A(j,j) \qquad \tilde{\tau}_j \quad \tau_j = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ \tilde{\tau}_j \end{pmatrix}$

$\quad A(j+1:n,\ j+1:n) \mathrel{-}= L(j+1:n,\ j)\, A(j,\ j+1:n) \qquad (I - \tau_j e_j^T)\, A^{(j)}$

$U = \mathrm{triu}(A)$



$u_{ij} = A_{ij} \quad j \geq i$

$u_{ij} = 0 \quad i < j$

$$\begin{matrix} \\ 1 \\ n-1 \end{matrix}\begin{pmatrix} \overset{1}{a_{11}} & \overset{n-1}{a_{12}^T} \\ a_{21} & A_{22} \end{pmatrix} = \begin{matrix} \\ 1 \\ n-1 \end{matrix}\begin{pmatrix} \overset{1}{1} & \overset{n-1}{0} \\ l_{21} & L_{22} \end{pmatrix} \begin{pmatrix} u_{11} & u_{12}^T \\ 0 & U_{22} \end{pmatrix}$$

$a_{11} = 1 \cdot u_{11} \Rightarrow u_{11} = a_{11}$

$a_{21} = l_{21} \cdot u_{11} \Rightarrow l_{21} = a_{21}/a_{11}$

$a_{12}^T = 1 \cdot u_{12}^T \Rightarrow u_{12}^T = a_{12}^T$

$$A_{22} = l_{21} \cdot u_{12}^T + L_{22} U_{22} = (a_{21}/a_{11}) \cdot a_{12}^T + L_{22} U_{22}$$

$$\Rightarrow A_{22} - (a_{21}/a_{11}) \cdot a_{12}^T = L_{22} U_{22}$$

Blocking

$$\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L_{11} & 0 \\ L_{21} & L_{22} \end{pmatrix} \begin{pmatrix} U_{11} & U_{12} \\ 0 & U_{22} \end{pmatrix}$$

$$A_{11} = L_{11} U_{11} \quad \checkmark$$

$$A_{21} = L_{21} U_{11} \Rightarrow L_{21} = A_{21} U_{11}^{-1}$$

$$A_{12} = L_{11} U_{12} \Rightarrow U_{12} = L_{11}^{-1} A_{12}$$

$$A_{22} = L_{21} U_{12} + L_{22} U_{22} = A_{21} U_{11}^{-1} L_{11}^{-1} A_{12} + L_{22} U_{22}$$

$$A_{22} - A_{21} A_{11}^{-1} A_{12} = L_{22} U_{22}$$

Schur complement

$$\begin{pmatrix} A_{11} & \cdots & A_{1b} \\ \vdots & & \\ A_{b1} & \cdots & A_{bb} \end{pmatrix} = \begin{pmatrix} L_{11} & & 0 \\ \vdots & L_{22} & \\ & & \ddots \\ L_{b1} & \cdots & L_{bb} \end{pmatrix} \begin{pmatrix} U_{11} & \cdots & U_{1b} \\ & U_{22} & \vdots \\ 0 & & \ddots \\ & & U_{bb} \end{pmatrix}$$

$$\boxed{A_{11} = L_{11} U_{11}} \qquad A(2:b, 2:b) \mathrel{-}= A(2:b, 1)\, A_{11}^{-1} A(1, 2:b)$$

$$\begin{pmatrix} L_{12} \\ \vdots \\ L_{b1} \end{pmatrix}$$