Feb 22, 2021

$$\underset{n\times n}{A}\;\underset{n\times 1}{x} = \underset{n\times 1}{b}$$

Example: kernel approx

$(z_1, y_1), \ldots, (z_n, y_n)$

$$y_i \approx \sum_{j=1} k(z_i, z_j)\, x_j$$

$$A_{ij} = k(z_i, z_j) \qquad Ax = y$$

$$Ax = b \Rightarrow x = A^{-1} b$$

(A^{-1}: just for notation)

~~inv(A)~~ $\cdot b$

bad
1. slower
2. not as stable
3. destroys structure

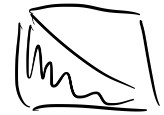
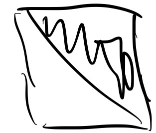
$$PA = LU$$

P: permutation

L:  1s on diag

U: 

$$Ax = b \Rightarrow PAx = Pb \Rightarrow LUx = Pb$$

1. $c = Pb$ (implicitly) $O(n)$
2. $Lw = c$  forward sub $O(n^2)$
3. $Ux = w$  backward sub $O(n^2)$

$$\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$$

$w_1 = 1,\; 2w_1 + w_2 = 1 \Rightarrow w_2 = -1$

$3w_1 + 2w_2 + w_3 = 3 \Rightarrow 3 - 2 + w_3 = 3$

$\Rightarrow w_3 = 2$

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w = 0
for i = 1:n
    s = c_i
    for j = 1:(i-1)
        s -= l_ij * w_j
    w_i = s / l_ii
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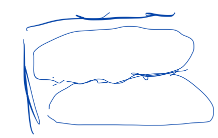
basic LU (no P)

$$G_{n-1} \cdots G_2 G_1 A = U$$

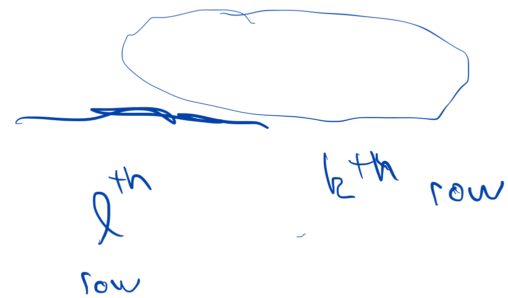
$$\underset{A}{\begin{pmatrix} x & x & x \\ x & x & x \\ x & x & x \end{pmatrix}} \rightarrow \underset{G_1 A}{\begin{pmatrix} x & x & x \\ 0 & x & x \\ 0 & x & x \end{pmatrix}} \rightarrow \underset{G_2 G_1 A}{\begin{pmatrix} x & x & x \\ 0 & x & x \\ 0 & 0 & x \end{pmatrix}}$$

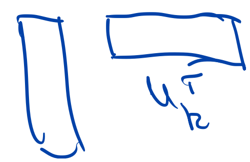


$U = A$

↓

return L, triu(U)





$L(k+1:n, k) = \tilde{\ell}_k$