Feb 12, 2021

Norms are how we measure length/size vectors, matrices

$\|\cdot\| : V \to \mathbb{R} \quad (V = \mathbb{R}^n)$

Length of a vector: $\|x\|$

difference: $\|x - y\| \Rightarrow$ distance / absolute error

Example: relative error $\frac{\|x-y\|}{\|x\|}$ dist to x / size of x

$\|\cdot\|$ is a norm if

(i) $\|x\| \geq 0, \quad \|x\| = 0 \iff x = 0$

(ii) $\|\alpha x\| = |\alpha| \, \|x\|$

(iii) $\|x + y\| \leq \|x\| + \|y\|$ sub-additivity / triangle inequality

$x, y \in V \quad (V = \mathbb{R}^n)$

Common vector norms

$$\|x\|_\infty = \max_i |x_i|$$ infinity norm, max norm

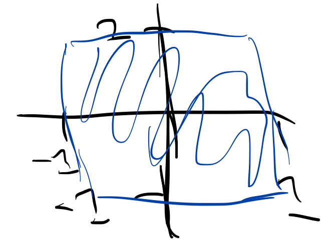
(i) $\|x\|_\infty \geq 0 \quad \|x\|_\infty = 0 \iff x = 0$

(ii) $\max_i |\alpha x_i| = \max_i |\alpha| |x_i| = |\alpha| \max_i |x_i|$

(iii) $\|x+y\|_\infty = \max_i |x_i + y_i| \leq \max_i |x_i| + |y_i|$

$\leq \max_i |x_i| + \max_i |y_i|$

$\|x\|_\infty \leq 1$



$$\|x\|_1 = \sum_{i=1}^n |x_i|$$ one norm

$\|x\|_1 \leq 1$

$$\|x\|_2 = \sqrt{\sum_{i=1}^n |x_i|^2}$$ two norm, Euclidean norm

$$x^T x = \sum_{i=1}^n x_i x_i = \|x\|_2^2$$