You may (and should) talk about problems with each other and with me, providing attribution for any good ideas you might get. Your final write-up should be your own.

1: Describing definiteness Suppose $A \in \mathbb{R}^{n \times n}$ is symmetric and positive definite. Give brief proofs of each of the following facts

- $A$ is invertible.
- $A^{-1}$ is positive definite.
- Any principal submatrix is positive definite.
- Any Schur complement is positive definite.

2: Beyond Cholesky Suppose $A \in \mathbb{R}^{n \times n}$ is symmetric and positive definite and $B \in \mathbb{R}^{n \times k}$ has full column rank (with $k < n$). Argue for the existence of the factorization

$$
\begin{bmatrix}
A & B \\
B^T & 0
\end{bmatrix} = \begin{bmatrix}
L_{11} & 0 \\
L_{21} & L_{22}
\end{bmatrix} \begin{bmatrix}
I & 0 \\
0 & -I
\end{bmatrix} \begin{bmatrix}
L_{11} & 0 \\
L_{21} & L_{22}
\end{bmatrix}^T
$$

and give a short code (using all the standard built-ins) to compute this factorization in the language of your choice.

3: Recover the map Suppose we are given $N$ pairs $(x^{(i)}, y^{(i)}) \in \mathbb{R}^n \times \mathbb{R}^n$. Write a short code (in any language) to solve

$$
\text{minimize} \sum_{i=1}^{N} \|y^{(i)} - Ax^{(i)}\|^2 \text{ over } A \in \mathbb{R}^{n \times n}
$$

You may use built-in functions for solving least squares. Illustrate your code on a sample set of points.