CS4220 Assignment-Based Review Questions

A1

P1 Assuming `x` and `y` are returned by `PointCloud`, give a vectorized fragment that computes the the number of points that are inside the largest circle that can be fit inside $\mathcal{E}(a,b)$.

P2 If a matrix can be written as the sum of N rank one matrices, then its rank is bounded above by N. Give an upper bound for the rank of the matrix behind the image that is displayed in the handout.

P3 If $A \in \mathbb{R}^{2\times 2}$ has a large 2-norm condition number, then it almoist sends a unit vector to the zero vector. Explain by a picture that graphically illustrates the SVD.

A2

P1 The set $R(a,b,\theta) = \{(a+\cos(\theta)t, b+\sin(\theta)t) : t \geq 0\}$ defines a ray. How could you discover if two different rays intersect?

P2 If F is a matrix, how would you compute F^{255}? Note that $255 = 1{+}2{+}4{+}8{+}16{+}32{+}64{+}128$.

P3 If $T \in \mathbb{R}^{n\times n}$ is upper triangular and $A = T^2$, how would you compute a_{ij} where $1 \leq i \leq j \leq n$?

A3

P1 Could the problem be solved if we computed $T - I = QR$ instead of $P(T-I) = LU$?

P2 How would you solve the problem if the (2,2) block in the matrix of coefficients was $-E$ where $E \in \mathbb{R}^{r\times r}$ is positive definite?

P3 Suppose $Q \in \mathbb{R}^{n\times n}$ is orthogonal. Note that $A = (Q+Q^T)/2 = (Q+Q^{-1})/2$ is symmetric. How would you compute the number of Q's eigenvalues that are in the left half plane?

A4

P1 Suppose $A \in \mathbb{R}^{2\times 1000}$ has 5 randomly located nonzeros in each of its two rows. Roughly how many nonzeros would you expect to be in the matrix GA where G is a rotation?

P2 Would Block Jacobi converge for this system if $A_1 = A_2 = I$ and $\| C \|_2 < 1$?

P3 Suppose $A \in \mathbb{R}^{m\times n}$ with $n < m$ and $v \in \mathbb{R}^m$. How would you compute the QR factorization of $[A \mid v]$ given that you have the QR factorization of A?

A5

P1 We wish to compute the SVD of n-by-n matrices $A_1, \ldots, A_p$ with the property that $\| A_i - A_{i+1} \|$ is very small for $i = 1{:}p-1$. Why would `ParJac` be of interest?

P2 It is known that T has one eigenvalue in each of the intervals $(0,1)$ and $(1,2)$. In the worst case, approximately how many times will `EigsToLeft` be called during the execution of `EigsInInterval(T,0,2)`?

P3 Describe a Newton-based method for computing the pth root of a positive number a.

P4 Suppose the derivative of f has exactly two zeros r_1 and r_2. Does it follow that `fzero(@f,[r1,r2])` returns a zero of f?

A6

P1 How would you compute an approximation to t_f?

P2 Given the availability of `DistE2Points` outline how `LSQNONLIN` could be used to find the best fitting ellipse to the given data.

P3 Graphically illustrate the shortcomings of steepest descent with exact line search when the level curves of the objective function are "cigar like".

A7

P1 The dimension of the given problem is five. Now suppose that the cost of red paint is the same as the cost of green paint. If effective use is made of `fminsearch`, then what would be the dimension of the problem?

P2 If small changes in the input data for a problem induce large changes in its solution, then we say that a problem is ill-conditioned. Using this language discuss what this problem is about.

P3 Nonlinear least squares is an important tool for parameter estimation. Explain.