CS4220 Assignment 6 (Corrected) Due: 4/18/13 (Thur) at 11pm

You must work either on your own or with one partner. You may discuss background issues and general solution strategies with others, but the solutions you submit must be the work of just you (and your partner). If you work with a partner, you and your partner must first register as a group in CMS and then submit your work as a group. Each problem is worth 5 points. One point may be deducted for poor style.

Topics: Golden section search, steepest descent, Newton's method, gradient, Hessian, line search, `fminbnd`, objective function

1 When Is Venus Brightest?

Assume that the following vectors specify the location of Venus and Earth at time t (in days):

$$V(t) = \begin{bmatrix} 66.01\cos(2\pi t/224.7) \\ 65.55\sin(2\pi t/224.7) \end{bmatrix}$$

$$E(t) = \begin{bmatrix} 94.5\cos(2\pi t/365.25) \\ 91.5\sin(2\pi t/365.25) \end{bmatrix}.$$

Assume that the Sun is at (0,0) (This is not a Keplerian set-up, but that is OK in this problem.) At time $t = 0$, Venus (V) is in between the Earth (E) and Sun (S) and is invisible. For a while, as t increases, the distance between the two planets

$$d(t) = \| V(t) - E(t) \|_2$$

increases and so does the fraction $F(t)$ of the illuminated portion of Venus as viewed from Earth. Indeed, that fraction is zero when angle S-V-E is 180 degrees, half when angle S-V-E is 90 degrees, and one when angle S-V-E is 0 degrees. Let t_f be the value of t when this last situation occurs for the first time. Using elementary trigonometry, it can be shown that

$$F(t) = \frac{1 + \cos(\mu(t)}{2}$$

where

$$\cos(\mu(t)) = \frac{V(t)^T(V(t) - E(t))}{\| V(t) \|_2 \| V(t) - E(t) \|_2}$$

To an observer on Earth, a case can be made that the brightness of Venus is given by

$$B(t) = \frac{F(t)}{d(t)^2}.$$

Even though $F(t)$ is increasing on $[0, t_f]$, eventually $B(t)$ begins to decrease as the inverse square law kicks in.

Write a function `P1()` that prints t_* and θ_* where $B(t_*)$ is the maximum value of $B(t)$ on the interval $[0, t_f]$ and θ_* is the S-V-E angle (in degrees) at time t_*. Use Golden Section search and be sure to justify your choice of the initial interval. (This can be done by supplying using a plot of $B(t)$.) Submit `P1` to CMS.

2 Distance of a Point Set to an Ellipse

Let $E(a, b, h, k, \phi)$ be the set of points $(x(t), y(t))$ defined as follows

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} h \\ k \end{bmatrix} + \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix} \begin{bmatrix} a\cos(t) \\ b\sin(t) \end{bmatrix} \qquad 0 \le t \le 2\pi$$

This is an ellipse with center (h, k), semiaxes a and b, and counter-clockwise tilt ϕ. For a given point (α, β), let $d(\alpha, \beta, a, b, h, k, \phi)$ be the minimum value of $(\alpha - x(t))^2 + (\beta - y(t))^2$ where $(x(t), y(t))$ is on the ellipse $E(a, b, h, k, \phi)$. Complete the following function so that it performs as specified:

```
  function z = DistE2Pts(u,v,a,b,h,k,phi)
% u and v are column n-vectors.
% z = d(u(1),v(1),a,b,h,k,phi) + ... + d(u(n),v(n),a,b,h,k,phi)
```

Thus, this function measures how far the point set $\{(u_1, v_1), \ldots, (u_n, v_n)\}$ is from the ellipse $E(a, b, h, k, \phi)$. Implement this function making effective use of `fminbnd`. Hint about the starting interval: the nearest point on $E(a, b, 0, 0, 0)$ to (α, β) is in the same quadrant as (α, β). A test script `P2()` is provided on the website. Submit your implementation of `DistE2Pts` to CMS.

3 Steepest Descent vs. Newton

Write a function `P3()` that demos the method of steepest descent with exact line search for finding the minimum of

$$f(x, y) \;=\; 100(y - x^2)^2 \;+\; (1 - x)^2.$$

`P3` should also demo the method of Newton applied to finding a zero of $\nabla f(x, y)$. For each method, output the iterates for three different starting values: $[\,-1\ 1\,]^T$, $[\,0\ 1\,]^T$, and $[\,2\ 1\,]^T$. You may use `fminbnd` for the exact line searches and you may use "\" for linear system solving. The loops that control the iterations should terminate after 20 steps or when $f(x, y) < .0001$, which ever comes first. Submit `P3` to CMS.