CS 4220: Some Review Problems

1. Given data $(t_1, y_1), \ldots, (t_m y_m)$, show how `fmninbnd` can be used to solve the problem of minimizing α_1, α_2 and λ so that

$$\phi(a, b, \lambda) \;=\; \sum_{i=1}^{m} \left(\alpha_1 + \alpha_2 e^{\lambda t_i} - y_i\right)^2 ; = \; \| \, A(\lambda)\alpha - y \, \|_2^2$$

is minimum?. What is $A(\lambda)$? What is the best choice of α given λ? What is the function passed to `fminbnd`?

2. Assume the availability of the following function:

```
  function [Q,R] = Update(Q0,R0,u,v)
% Q0 is m-by-m orthogonal, R0 is m-by-n upper triangular, u is m-by-1, v is n-by-1
% Q is m-by-m orthogonal, R is m-by-n upper triangular, Q*R = Q0*R0 + u*v'
```

(a) How would you solve min $\| \, Ax - b \, \|$ and min$\| \, \tilde{A}x - b \, \|$ given that $\tilde{A}$ is defined by $\tilde{A}(:,k) = A(:,k) - sum(A(:,k))$.

(b) How would you solve min $\| \, Ax - b \, \|$ and min$\| \, \tilde{A}x - b \, \|$ given that $\tilde{A}$ is A with a new row appended to the bottom.

(c) How would you solve min $\| \, Ax - b \, \|$ and min$\| \, \tilde{A}x - b \, \|$ given that $\tilde{A}$ is A with its first column replaced by a given vector.

3, $M \in \mathbb{R}^{n \times n}$ has the property that the first $n-1$ columns of $M - I_n$ are independent. How would you compute a nonzero vector x so $Mx = x$ given that $P(M - I) = LU$ is on tap?

4. $G \in \mathbb{R}^{n \times n}$ is a 0-1 matrix in sparse format. Define the matrix $B \in \mathbb{R}^{n \times n}$ by

$$B(:,k) \;=\; \begin{cases} G(:,k)/sum(G(:,k)) & \text{if } G(:,k) \neq 0 \\ \\ \text{ones}(n,1)/n & \text{otherwise} \end{cases}$$

Let $0 < \rho < 1$ be a scalar. How would you apply the power method to compute the dominant eigenvector for the matrix $A = \rho B + (1-\rho) * \text{ones}(n,n)/n$. Show carefully how you would organize the matrix-vector products.

5. $Q \in \mathbb{R}^{n \times n}$ is orthogonal with a unique eigenvalue equal to one. What happens if the power method is applied to the matrix $A = I + (Q + Q^T)/2$? Hint: Draw a picture of A's eigenvalues.

6. (a) How would you minimize $\| \, Ax \, \|_2$ subject to the constraint that $x_1 = 1$.? (b) Given $t \in \mathbb{R}^m$, how would you determine a scalar τ so that

$$\sum_{k=1}^{m} [\sin(t_i - \tau)]^2$$

is minimized? Hint: Trig identities and SVD.

7. What does a step of steepest-descent with exact line search look like when applied to the minimization of $\phi(x) = x^T Ax / x^T x$ where $A \in \mathbb{R}^{n \times n}$ is symmetric?

8. Newton's method is applied to find a, b, and c so that $(x-a)(x-b)(x-c) = x^3 + 5x^2 - 3x + 4$. Describe the linear system that must be solved each step.

9. How could Householder tridiagonalization be used to solve the linear system $(A^2 - 5A + 6I)x = b$ where $A \in \mathbb{R}^{n \times n}$ is symmetric?

10. Suppose $A, B \in \mathbb{R}^{m \times n}$ and $m < n/2$. How could you find $x \in \mathbb{R}^n$ so that $Ax = b$ and $Bx = c$ where $b, c \in \mathbb{R}^m$ are given?