

CS 4220: Assignment 6

Due: Monday, April 26, 2010 (In Class or in Upson 5153 by 5pm).

Scoring for each problem is on a 0-to-5 scale (5 = complete success, 4 = overlooked a small detail, 3 = good start, 2 = right idea, 1 = germ of the right idea, 0 = missed the point of the problem.) One point will be deducted for insufficiently commented code. Unless otherwise stated, you are expected to utilize fully MATLAB's vectorizing capability subject to the constraint of being flop-efficient. Test drivers and related material are posted on the course website <http://www.cs.cornell.edu/courses/cs4220/2010sp/>. For each problem submit output and a listing of all scripts/functions that *you* had to write in order to produce the output. You are allowed to discuss *background* issues with other students, but the codes you submit must be your own.

P1. (Supernovae Luminosity)

Type I supernovae have been found to follow a specific pattern of luminosity (brightness). Beginning a few days after the maximum luminosity this pattern may be described by

$$L(t) = C_1 \exp(-t/\alpha_1) + C_2 \exp(-t/\alpha_2)$$

where t is the time in days after the max and $L(t)$ is the luminosity at time t . The function `[t,Lvals] = LuminData()` returns the actual data associated with the supernovae SN1939A. In particular, `Lvals(i)` is the luminosity at time `t(i)` for `i=1:21`.

By making effective use of MATLAB's nonlinear least squares solver `LSQNONLIN`, produce estimates of C_1 , C_2 , α_1 and α_2 so that

$$\phi(C_1, C_2, \alpha_1, \alpha_2) = \sum_{i=1}^{21} (L(t_i) - \text{Lvals}(i))^2$$

is minimum. Strive for five significant digits of accuracy. Submit a listing of your solution script and the function that you hand over to `LSQNONLIN` and a plot that displays the optimal fitting function and the data. Indicate the associated values of the four parameters in the title of the plot. Include comments in your code about how you determined appropriate starting values and what happens if the starting value choice is poor.

P2. (Mars from Earth According to Ptolemy)

Assume the availability of functions `[xE,yE] = Earth(t)` and `[xM,yM] = Mars(t)` that return the "exact" xy coordinates of Earth and Mars at time t (days). Assume that the Sun is positioned at (0,0) and that at time $t = 0$, y_E and y_M are zero and that $0 < x_E < x_M$. In other words, the two planets are lined up with the Sun and are situated on the positive x -axis with Mars further out.

Your job is to determine epicycle parameters P_1 , r_2 , P_2 , r_3 , and P_3 so that if

$$x(t) = \cos(2\pi t/P_1) + r_2 \cos(2\pi t/P_2) + r_3 \cos(2\pi t/P_3)$$

$$y(t) = \sin(2\pi t/P_1) + r_2 \sin(2\pi t/P_2) + r_3 \sin(2\pi t/P_3)$$

then the direction of the vector

$$Q(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$$

is as close as possible to the direction of the vector

$$R(t) = \begin{bmatrix} x_M(t) - x_E(t) \\ y_M(t) - y_E(t) \end{bmatrix}$$

over the time interval $[0, 2000]$.

In this 3-circle epicycle model, think of the Earth at the center of a unit disk whose period of revolution is P_1 . Centered on the rim of this disk is a second disk whose radius is r_2 and whose period of revolution is P_2 . Finally, centered on the rim of the second disk is a third disk whose radius is r_3 and whose period of revolution is P_3 . On the rim of the third disk is Mars and so $Q(t)$ is the vector that points to Mars from the observation point $(0, 0)$.

For the objective function use

$$\phi(P_1, r_1, P_2, r_3, P_3) = \sum_{i=1}^{2000} \sin(\theta_i)^2$$

where θ_i is the angle between the two vectors $Q(i)$ and $R(i)$. Recall that vector cross products can be used to compute the sine of the angle between two vectors. Make effective use of `fminsearch`. Getting a good initial guess is important. It might be handy to use the fact that the two planets have approximate periods 365 and 697 (days) and that the average Sun-Mars distance is about 1.524 times the average Sun-Earth distance. Note that if the r and P value for a disk have the same sign, then it is rotating counter-clockwise. If the r and P value have opposite sign, then rotation is clockwise.

Submit all the code that you use to solve this problem together with output and (as necessary) written comments that explain how you went about getting the solution.

P3. (Intersecting Ellipses)

We say that the ellipse \mathcal{E}_1 and the ellipse \mathcal{E}_2 satisfy Property X if neither intersects the line that goes through the other's two foci. Assume that if h, k, a, b , and τ are the values of `E.h`, `E.k`, `E.a`, `E.b`, and `E.tau`, then E represents the ellipse

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} h \\ k \end{bmatrix} + \begin{bmatrix} \cos(\tau) & -\sin(\tau) \\ \sin(\tau) & \cos(\tau) \end{bmatrix} \begin{bmatrix} a \cos(t) \\ b \sin(t) \end{bmatrix}$$

Implement the following function:

```
function [x,y] = Intersection(E1,E2)
% E1 is an ellipse and E2 is an ellipse that satisfy Property X.
% x and y are column vectors that house their (possible) intersection points.
% In particular, if E1 and E2 fail to intersect, then x and y are empty.
% If x and y intersect in two points, then those points are given by (x(1),y(1))
% and (x(2),y(2)). (Ignore the situation where E1 and E2 are tangent to each other.)
```

Make effective use of `fsolve` and compute the intersection points to 6 significant digits. Submit listing and output when the script P3 is applied.

Ellipse facts can be found at <http://www.cs.cornell.edu/cv/OtherPdf/Ellipse.pdf>.