CS 4220: Assignment 4 Hints

P1. (Skew-Symmetric Tridiagonalization)

Remember that $z^T A z = 0$ if $A = -A^T$. This implies that a skew-symmetric matrix has a zero diagonal.

The A matrix looks like this at the start of step k

$$A = \begin{bmatrix} 0 & -\alpha_1 & 0 & 0 & 0 & 0 & 0 \\ \alpha_1 & 0 & -\alpha_2 & 0 & 0 & 0 & 0 \\ 0 & \alpha_2 & 0 & x & x & x & x \\ 0 & 0 & x & 0 & c & c & c \\ 0 & 0 & x & c & 0 & c & c \\ 0 & 0 & x & c & c & 0 & c \\ 0 & 0 & x & c & c & c & 0 \end{bmatrix} \qquad (n = 7, k = 3)$$

You then determine a Householder matrix:

$$(I - 2vv^T) \begin{bmatrix} x \\ x \\ x \\ x \end{bmatrix} = \begin{bmatrix} \alpha_3 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

and update (exploiting skew-symmetry):

$$\begin{bmatrix} 0 & x & x & x \\ x & 0 & c & c \\ x & c & 0 & c \\ x & c & c & 0 \end{bmatrix} \leftarrow (I - 2vv^T) \begin{bmatrix} 0 & c & c & c \\ c & 0 & c & c \\ c & c & 0 & c \\ c & c & c & 0 \end{bmatrix} (I - 2vv^T)$$

taking you to the beginning of the next step:

$$A = \begin{bmatrix} 0 & -\alpha_1 & 0 & 0 & 0 & 0 & 0 \\ \alpha_1 & 0 & -\alpha_2 & 0 & 0 & 0 & 0 \\ 0 & \alpha_2 & 0 & -\alpha_3 & 0 & 0 & 0 \\ 0 & 0 & \alpha_3 & 0 & x & x & x \\ 0 & 0 & 0 & x & 0 & c & c \\ 0 & 0 & 0 & x & c & 0 & c \\ 0 & 0 & 0 & x & c & c & 0 \end{bmatrix} \qquad (n = 7, k = 4)$$

P2. (Special Schur Decomposition)

What does $Q^T A Q$ look like if $X = QR$?

P3. (A Rank-1 Adjustment Eigenproblem)

Use the Schur decomposition of A to transform the power method matrix to the form diagonal-plus-rank-1. The matrix-vector products then become $O(n)$.

When the power method is applied to a matrix C, if z is the current unit 2-norm iterate and $y = Cz$, then $\mu = y^T z$ is the associated eigenvalue estimate. This choice minimizes $\| Cz - \mu z \|_2$.

P4. (A Constrained Max Problem)

Suppose

$$Q \;=\; \left[\; q \;|\; Q_2 \;\right]$$

where $q \in \mathbb{R}^n$ and $Q_2 \in \mathbb{R}^{n \times n-1}$. If $A \in \mathbb{R}^{n \times n}$ then

$$Q^TAQ \;=\; \left[\begin{array}{cc} q^TAQ & q^TAQ_2 \\ Q_2^TAq & Q_2^TAQ_2 \end{array}\right]$$