CS 4220: Assignment 1

Due: Wednesday, February 6, 2008 (In Lecture)

Scoring for each problem is on a 0-to-5 scale (5 = complete success, 4 = overlooked a small detail, 3 = good start, 2 = right idea, 1 = germ of the right idea, 0 = missed the point of the problem.) One point will be deducted for insufficiently commented code. Unless otherwise stated, you are expected to utilize fully MATLAB'S vectorizing capability subject to the constraint of being flop-efficient. Test drivers and related material are posted on the course website `http://www.cs.cornell.edu/courses/cs4220/2009sp/`. For each problem submit output and a listing of all scripts/functions that *you* had to write in order to produce the output. You are allowed to discuss *background* issues with other students, but the codes you submit must be your own.

P1. (Floating Point Representation)

Assume that p is an integer that satisfies $1 \le p \le 62$. *Positive machine numbers* have the form

$$x = \left(1 + \sum_{i=1}^{p-1} \frac{b_i}{2^i}\right) \times 2^E$$

where each b_i is either 0 or 1 and E is an integer that satisfies

$$-(2^{63-p} - 2) \ \le \ E \le \ (2^{63-p} - 1)$$

If $p = 53$ then this is essentially IEEE double precision format. For a given p, let $\phi(p)$ be the largest integer k such that 10^k is a machine number. Write a MATLAB script that prints p and $\phi(p)$ for $p = 1{:}62$. Submit output and the script that generated it. Include enough comments in the script so that I can reconstruct your reasoning. Good time to review basic MATLAB constructs: `for, while, if, log10, log2, find, ...` .

P2. (Matrix Formation)

Complete the following function so that it performs as advertised

```
  function Q = Rank1SeqProd(V)
% V is an m-by-p lower triangular matrix with p<m
% Q = V_1*V_2*...*V_p where V_k = eye(m,m) - V(:,k)*V(:,k)'
```

Submit a listing of your solution and the output produced when the test script `P2` is applied.

P3. (Convergence of Matrix Powers)

Suppose $T \in \mathbb{R}^{n\times n}$ is upper triangular and that $0 < \rho < 1$. If $|t_{ii}| < \rho$, then it can be shown that $T^k \to 0$. The rate of convergence depends on the value of ρ and the norm of the strictly upper triangular portion of T. Write a paragraph that explains this in more detail. How does it depend upon ρ and the elements above the diagonal? Base your answer upon careful experimentation. You may include one plot to go along with the paragraph you submit. Submit a listing of the codes involved in your experimentation.

P4. (Structured Linear Equation Solve)

Define the n-by-n exchange matrix E_n by $E_n = I_n(:, n{:} -1{:}1)$, i.e., the identity with its column order reversed. Complete the following function so that it performs as specified,

```
 function x = StructuredSolve(A,b)
% A is a 2n-by-2n symmetric positive definite matrix that satisfies EAE = A
% where E is the 2n-by-2n exchange matrix.
% b is a 2n-by-1 vector that satisfies b = Eb.
% x solves Ax = b
```

Submit a listing of your solution and the output produced when the test script P4 is applied. Hint: You can get a handle on the underlying structure by looking at $Ax = b$ as a 2-by-2 block system.

P5. (Rate of Change)

Suppose $A \in \mathbb{R}^{n \times n}$ is nonsingular and that $E \in \mathbb{R}^{n \times n}$ and $b \in \mathbb{R}^n$. For small values of t, the linear system $(A + tE)x = b$ is nonsingular and the components of $x = x(t)$ are differentiable. Complete the following function so that it performs as specified

```
  function alfa = RateOfChange(A,E,b,k)
 % A and E are n-by-n with A nonsingular.
 % b is n-by-1 and k is an integer that satisfies 1<=k<=n
 % Let x(t) be the solution to (A+tE)x  = b.
 % alfa is the rate of change at t = 0 of the k-th component of x(t)
```

Make effective use of MATLAB's lu function. Hint: Differentiate the equation $(A + tE)x(t) = b$ with respect to t and set $t = 0$ in the result. Then manipulate to get an expression for the desired rate of change.

Submit a listing of your solution and the output produced when the test script P5 is applied.