CS 4220: Assignment 3

Due: Wednesday, March 4, 2009 (In Lecture)

Scoring for each problem is on a 0-to-5 scale (5 = complete success, 4 = overlooked a small detail, 3 = good start, 2 = right idea, 1 = germ of the right idea, 0 = missed the point of the problem.) One point will be deducted for insufficiently commented code. Unless otherwise stated, you are expected to utilize fully MATLAB'S vectorizing capability subject to the constraint of being flop-efficient. Test drivers and related material are posted on the course website `http://www.cs.cornell.edu/courses/cs4220/2009sp/`. For each problem submit output and a listing of all scripts/functions that *you* had to write in order to produce the output. You are allowed to discuss *background* issues with other students, but the codes you submit must be your own.

P1. (Fitting a Conic)

We have a cloud of points in the plane $(x_1, y_1), \ldots, (x_m, y_m)$ that we want to fit with a conic, i.e., ellipse, hyperbola, parabola. The general formula for a conic is given by

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

If $B^2 \leq 4AC$ then the conic is an ellipse. Note that the problem of minimizing

$$\phi(A, B, C, D, E, F) \;=\; \sum_{i=1}^{m} \left(Ax_i^2 + Bx_iy_i + Cy_i^2 + Dx_i + Ey_i + F\right)^2$$

is a linear least squares problem. Refer to the optimizing conic as the "best conic fit". Complete the following function so that it performs as specified:

```
  function [A,B,C,D,E,F,q] = IsAnEllips(x,y)
% x and y are column m-vectors.
% q is the largest integer so that the best conic fit of the data
% (x(1),y(1)),...,(x(q),y(q)) is an ellipse. A, B, C, D, E, and F are
% the coefficients of the ellipse. If no such q exists, then q is
% assigned the value of zero.
```

You are allowed to use the MATLAB functions `qr`. Test your implementation with the script `A3P1`. Submit output and listing.

P2. (A Pair of Underdetermined Systems)

Suppose $A \in \mathbb{R}^{m\times n}$ has rank $m < n$ and that we have a solution to x_c to the underdetermined linear system $Ax_c = b$ where $b \in \mathbb{R}^m$ is given. If the columns of $N \in \mathbb{R}^{m\times n-m}$ span the null space of A, then any solution to $Ax = b$ has the form $x_c + Nz$ where $z \in \mathbb{R}^{n-m}$. A particular solution to $Ax = b$ and a null space matrix N is easily obtained from the QR factorization of A^T.

Complete the following function so that it performs as specified:

```
  function [x1,x2] = Under2(A1,b1,A2,b2)
% A1 is m1-by-n with rank(A1) = m1 < n, b1 is m1-by-1
% A2 is m2-by-n with rank(A2) = m2 < n, b2 is m2-by-1
% A1*x1 = b1 and A2*x2 = b2 and no other pair of solutions to these
% two underdetermined systems are closer to each other in 2-norm. That
% is, if A1*w1 = b1 and A2*w2 = b2, then norm(x1-x2,2) <= norm(w1-w2,2).
```

You are allowed to use the MATLAB functions `qr` and `svd`. Test your implementation with the script `A3P2`. Submit output and listing.

P3. (Rotate)

Complete the following function so that it performs as specified

```
  function [c,s] = Rotate(x,y)
% x and y are column n-vectors.
% c and s are a cosine-sine pair with the property that the columns
% of [x y]*[c s; -s c] are orthogonal to each other.
```

This is an SVD problem. Test your implementation with the script `A3P3`. Submit output and listing.

P4. (Low Rank SVD)

Complete the following function so that it performs as specified.

```
  function s = SVD3(Y,Z)
% Y and Z are m-by-3 matrices with full column rank.  Assume m >> 3.
% s is a column 3-vector where s(1)>=s(2)>=s(3) are the three positive
% singular values of YZ'
```

Test your implementation with the script `A3P4`. Submit output and listing.