CS 421: Assignment 4 Extra Credit Variants

Due: Friday, October 27, 2006 (In Lecture)

You must turn in P1 by Monday October 23 (in lecture) if you want to submit P1X. You must turn in P2 by Monday October 23 (in lecture) if you want to submit P2X.

P1X. (Finding a Specific Eigenpair of an Orthogonal Matrix)

Suppose $U \in \mathbb{R}^{n\times n}$ is orthogonal and has eigenvalues $e^{i\theta_1}, \ldots, e^{i\theta_n}$ with $0 \le \theta_1 < \theta_2 < \cdots < \theta_n < 2\pi$. Given $0 \le \phi < 2\pi$ we wish to compute $\lambda = e^{i\theta_*}$ and a unit 2-norm $v \in \mathbb{C}^n$ so that $Uv = \lambda v$ where λ is the closest eigenvalue of U to $e^{i\phi}$. Notice that if we set $\phi = 0$ we have (basically) problem P1. Write a generalization of the function BigEvec of the form `[v,lambda] = OrthogEvec(U,tol,phi)` that computes approximations to λ and v using the power method with termination criteria `norm(U*v - lambda*v,2) <= tol`. Hint: What are the eigenvalues of $U + \alpha e^{i\phi} I_n$? Justify the mathematics behind your implementation. A test script will be placed on the website. Submit your implementation and the output.

P2X. (Limiting the Search for the Minimum Value of $\sigma_n(\mu)$)

In P2 you are asked to compute the minimum value of the function $\sigma_n(\mu)$ which is defined to be the smallest singular value of the matrix $A + i\mu I_n$. This is a nonlinear optimization problem with no closed form solution. To get an estimate of the minimum we lowered our aim by simply minimizing this function on the points `linspace(-m,m,1000)` where `m = max(abs(eig(A)))`. I can't prove that the actual minimum value of $\sigma_n(\mu)$ occurs on the real interval $[-m, m]$. That would be a nice justification for our chosen sampling points. However, it can be shown that $\sigma_n(\mu)$ attains its minimum value on the interval $[-\| A \|_2 \,,\ +\| A \|_2\,]$. Prove this. Hint: Show that for any unit vector $v \in \mathbb{C}^n$, the function $f(\mu) = \| (A + \mu i I_n)v \|_2^2$ is monotone increasing if $\mu \ge \| A \|_2$. Proceed to show that $\sigma_n(\mu)$ is monotone increasing on $[\| A \|_2, \infty)$ and (by symmetry) monotone decreasing on $(-\infty, -\| A \|_2]$. One page max!