CS 421: Numerical Analysis
Fall 2004
Problem Set 1

Handed out: Wed., Sep. 8.

Due: Fri., Sep. 17 in lecture.

1. Let M be an $n \times n$ *elementary* unit lower triangular matrix, that is, a matrix of the form $I - \mathbf{m}\mathbf{e}_k^T$ where $\mathbf{m} \in \mathbf{R}^n$ is a vector whose first k entries are 0's and $\mathbf{e}_k$ is the kth column of the identity matrix. See p. 67 of the text for an example and more explanation. Let $P(i,j)$ be the permutation matrix that exchanges row i with row j, but leaves other rows unchanged. Assume $i > k$ and $j > k$. Show that $P(i,j)M = NP(i,j)$, where N is some other elementary lower triangular matrix. Exactly how is N related to M?

2. (a) Forward substitution presented in Algorithm 2.1 of the text is based on saxpy ("saxpy" stands for "scalar a [times] x plus y") instead of inner product. Rewrite this version of forward substitution as a Matlab fragment, and be sure to vectorize the inner loop. Saxpy is preferable to inner product on some parallel and vectorized hardware architectures.

   (b) Can matrix-vector multiplication be written so that its inner loop is a saxpy? an inner product? How about plain Gaussian elimination? Explain your answers.

3. In lecture, a Matlab fragment for GEPP was provided that computes an array `p` to store information about the row exchanges. Write (on paper) a Matlab fragment that takes as input the array `p` and produces as output the $n \times n$ corresponding permutation matrix P.

4. Show that Heath's algorithm for GEPP applied to a singular matrix described on p. 71 can be modified to produce an infinite number of factorizations $PA = LU$, where P is a permutation matrix, A is a singular matrix, L is unit lower triangular with all multipliers in $[-1, 1]$, and U is upper triangular. The existence of an infinite number of factorizations may depend on which diagonal entry of U is zero.

   (Note: Heath explains on p. 78 that LU factorization of PA is unique, but his argument requires the assumption that PA is nonsingular.)