CS 421: Numerical Analysis
Fall 2002
Problem Set 1

Handed out: Wed., Sep. 11.

Due: Fri., Sep. 20 in lecture.

1. Let M be an $n \times n$ *elementary* unit lower triangular matrix, that is, a matrix of the form $I - \mathbf{m}\mathbf{e}_k^T$ where $\mathbf{m} \in \mathbf{R}^n$ is a vector whose first k entries are 0's and $\mathbf{e}_k$ is the kth column of the identity matrix. See p. 67 of the text for an example and more explanation. Let $P(i, j)$ be the permutation matrix that exchanges row i with row j, but leaves other rows unchanged. Assume $i > k$ and $j > k$. Show that $P(i, j)M = NP(i, j)$, where N is some other elementary lower triangular matrix. Exactly how is N related to M?

2. Let M be an $n \times n$ elementary unit lower triangular matrix $I - \mathbf{m}\mathbf{e}_k^T$ such that all entries of $\mathbf{m}$ have absolute value at most 1. Consider solving $M\mathbf{x} = \mathbf{b}$ for $\mathbf{x}$. Show that the absolute values of entries in $\mathbf{x}$ are all no more than twice the maximum absolute value in $\mathbf{b}$, i.e.,
$$\max_i |x(i)| \leq 2 \max_i |b(i)|.$$

3. In lecture, a Matlab fragment for GEPP was provided that computes an array `p` to store information about the row exchanges. Write (on paper) a Matlab fragment that takes as input the array `p` and produces as output the $n \times n$ corresponding permutation matrix P.

4. Show that the product of two $n \times n$ lower triangular matrices is itself lower triangular. Provide a Matlab fragment for multiplying two $n \times n$ lower triangular matrices that avoids unnecessary operations on zeros. The innermost loop should be vectorized. Analyze the number of flops required by your fragment, accurate to the leading term.