CS 421: Numerical Analysis
Fall 2002
**Practice Prelim 1**

Handed out: Mon., Sep. 23.

This was a timed 75-minute exam given in 2000. All the questions were weighted equally. Write your answers in the exam booklet. This test is closed-book and closed-note, but students were allowed to consult a 8.5-by-11 sheet of paper prepared in advance.

1. Write down an example of a $2 \times 2$ ill-conditioned linear system $A\mathbf{x} = \mathbf{b}$. Write down an example of a $2 \times 2$ well-conditioned linear system.

2. A "checkerboard" matrix $A$ has the property that $A(i,j) = 0$ whenever $i + j$ is odd. Let $A, B$ be two $n \times n$ checkerboard matrices. How many flops, accurate to the leading term, are required for computing the product $AB$?

3. Let $A$ be a unit lower triangular matrix. Consider performing plain Gaussian elimination on $A$. (a) Show that the factorization $A = LU$ that would be computed by plain Gaussian elimination can in fact be computed without any flops using a fairly trivial algorithm for this special case. (b) Is plain Gaussian elimination followed by forward and back substitution a stable algorithm for solving $A\mathbf{x} = \mathbf{b}$ for this special case of $A$? [Hint for (b): consider $\|L\|_\infty \cdot \|U\|_\infty$ versus $\|A\|_\infty$.]

4. Consider the scalar function $f(x) = 1/x$. Show that this function is well conditioned, i.e., show that a small relative perturbation to the data (that is, $x$) results in a small relative perturbation to the function. Your analysis should be valid for any nonzero data.

5. Prove the following inequality, which is valid for all $\mathbf{x} \in \mathbf{R}^n$:

$$\|\mathbf{x}\|_2 \leq (\|\mathbf{x}\|_1 \cdot \|\mathbf{x}\|_\infty)^{1/2}.$$