CS 421 – **Problem Set 5**
Due: Wednesday, November 17, 1999

1. Let $x \in \Re^n$. Define $Q = I - \frac{2uu^T}{u^Tu}$ where $u = x + sgn(x_1)\|x\|e_1$. Show that $Qx = sgn(x_1)\|x\|e_1$.

2. In class we discussed the computation of $A = QR$ via Householder transformations. How many flops are required by this approach (high-order term including the coefficient) when Q is explictly formed as a square matrix versus the case when the Householder transformations that form Q are not multiplied together?

3. Support or refute: The number of flops required to solve a square linear system $Ax = b$ via a sequence of Householder transformations (and a backsolve) is about twice that required by the LU-factorization approach (with partial pivoting).

4. Let A be a sparse matrix of full column rank. The George-Heath technique (discussed in class) attempts to compute a sparse triangular factor R, under orthogonal transformations, by first permuting the columns of matrix A. Show that row permutations will not affect the sparsity of R. Are row permutations of any use in this setting?

5. A direct sparse finite-difference method (based on a coloring of the intersection graph) for determining a sparse Jacobian matrix can be viewed as a diagonal system of equations. Illustrate the system for the general case.

6. Consider $\min\{q(x) = c^Tx + \frac{1}{2}x^THx : Ax = b\}$ where H is a symmetric matrix of order n, c is an n-vector, A is an m-by-n matrix of rank m, and b is an m-vector.

   (a) Show how to compute a matrix Z whose columns form an orthonormal basis for the null space of A.

   (b) Show how Z can be used to compute the solution this constrained quadratic minimization problem. Comment on the uniqueness and existence of solutions: properties of H should play a role in your discussion.

7. Using the MATLAB Optimization Toolbox, or other optimization software, solve the following minimization problem.

$$\min f(x) = \sum_{i=1}^{n-1} [100(x_{i+1} - x_i^2)^2 + (1 - x_i)^2].$$

Try different dimensions,$n = 2, n = 6, n = 10, n = 100$ with $x = 0$ the initial starting point. If possible try different algorithmic options and even different optimization methods.

Note that the solution is $x = (1, 1, ..., 1)$ with function value zero. Compare and discuss your results.

For the $n = 2$ case plot a contour map and the trajectories followed by the different approaches. (Note: There are other local minima and stationary points floating around, so don't be too surprised at convergence to the "wrong" point.)