CS 421 – **Problem Set 5**
Due: Wednesday, November 17, 1999

1. Let $x \in \Re^n$. Define $Q = I - \frac{2uu^T}{u^T u}$ where $u = x + sgn(x_1)\|x\|e_1$. Show that $Qx = sgn(x_1)\|x\|e_1$.

2. In class we discussed the computation of $A = QR$ via Householder transformations. How many flops are required by this approach (high-order term including the coefficient) when $Q$ is explictly formed as a square matrix versus the case when the Householder transformations that form $Q$ are not multiplied together?

3. Support or refute: The number of flops required to solve a square linear system $Ax = b$ via a sequence of Householder transformations (and a backsolve) is about twice that required by the $LU$-factorization approach (with partial pivoting).

4. Let $A$ be a sparse matrix of full column rank. The George-Heath technique (discussed in class) attempts to compute a sparse triangular factor $R$, under orthogonal transformations, by first permuting the columns of matrix $A$. Show that row permutations will not affect the sparsity of $R$. Are row permutations of any use in this setting?

5. A direct sparse finite-difference method (based on a coloring of the intersection graph) for determining a sparse Jacobian matrix can be viewed as a diagonal system of equations. Illustrate the system for the general case.

6. Consider $\min\{q(x) = c^T x + \frac{1}{2}x^T Hx : Ax = b\}$ where $H$ is a symmetric matrix of order $n$, $c$ is an $n$-vector, $A$ is an $m$-by-$n$ matrix of rank $m$, and $b$ is an $m$-vector.

   (a) Show how to compute a matrix $Z$ whose columns form an orthonormal basis for the null space of $A$.

   (b) Show how $Z$ can be used to compute the solution this constrained quadratic minimization problem. Comment on the uniqueness and existence of solutions: properties of $H$ should play a role in your discussion.

7. Using the MATLAB Optimization Toolbox, or other optimization software, solve the following minimization problem.

$$\min f(x) = \sum_{i=1}^{n-1} [100(x_{i+1} - x_i^2)^2 + (1 - x_i)^2].$$

Try different dimensions, $n = 2, n = 6, n = 10, n = 100$ with $x = 0$ the initial starting point. If possible try different algorithmic options and even different optimization methods.

Note that the solution is $x = (1, 1, ..., 1)$ with function value zero. Compare and discuss your results.

For the $n = 2$ case plot a contour map and the trajectories followed by the different approaches. (Note: There are other local minima and stationary points floating around, so don't be too surprised at convergence to the "wrong" point.)