CS 421 – **Problem Set 3**
Due: Friday, October 15, 1999

1. Let A be an m-by-n matrix. Write a Matlab program, without using the Matlab "norm" function, to compute both $\|A\|_1$ and $\|A\|_\infty$. Test your program on a few examples comparing your answer to that provided by the appropriate versions of the Matlab "norm" function.

2. Illustrate an effective envelope Cholesky factorization algorithm (you do **not** have to implement it in Matlab).

3. (a) Write an efficient Matlab program to compute the Cholesky factor of a *banded* symmetric positive definite matrix A:

   function[BL] = bchol(BA).

   The entries of the lower half of the banded symmetric positive definite matrix A, with bandwidth β, should be stored in the $(\beta + 1)$-by-n array BA (A is never explicitly computed or stored.) The $(\beta + 1)$-by-n array BL contains the nonzero entries of the Cholesky factor of A.

   (b) Either modify your triangular solve routines from the previous assignment, or write new routines, to solve

   $$Ly = b, \quad L^T x = y$$

   where L is the banded Cholesky factor stored in BL. The full matrix L should never be explicitly computed or stored.

   (c) Test you programs on different systems $Ax = b$: $\beta = 0, 1, 3, 5, 8, 10, n$ and $n = 20, 25$ (14 test problems in total). You can generate suitable test problems in reverse: first use "rand" to generate BL and b and then compute BA (a program must be written). Then, to solve $Ax = b$ use *bchol* to obtain BL, followed by your triangular solves. In each case compute the relative residual, $\frac{\|Ax-b\|}{\|A\|\cdot\|x\|}$, where x is the computed solution. You may assume that $\|A\| = 1$. How many flops does the factorization part of your algorithm require? (I am interested in the high-order term, including the coefficient.)

4. (a) Let r be a known scalar and $s \neq 0$ a known vector in $\Re^n$. What is a solution to the problem

$$\min_v \{\|v\|_2 : v^T s = r\}?$$

[Hint: Any vector v can be written, $v = \alpha s + w$ where α is a scalar and $s^T w = 0$.]

(b) Using part (a), justify the Broyden (or secant) update which solves the problem $min_E \, \{\|E\|_F : \; Es = r\}$ for known vectors s and r.