3D Viewing

CS 417 Lecture 16

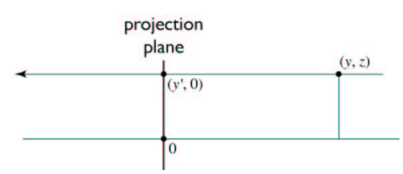
Announcements

- Prelim 1 tomorrow
  - Covers topics from Homeworks 1 & 2
    - that is, lectures 1–8
    - minus Fourier transforms
      (but including sampling in general)
  - One-page reference sheet allowed

Mathematics of projection

- Assume eye point at 0 and plane perpendicular to z
- Parallel case
  - a simple projection: just toss out z
- Perspective case: scale diminishes with z
  - and increases with *d*

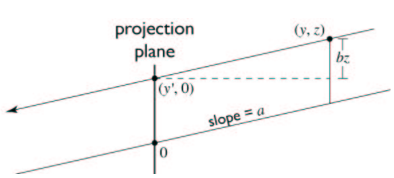
Parallel projection: orthographic



to implement orthographic, just toss out z:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

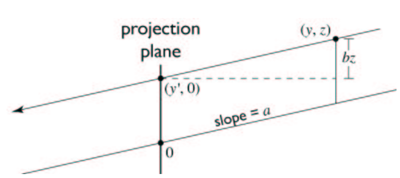
Parallel projection: oblique



to implement oblique, rewrite in oblique basis, then toss out z:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} x - az \\ y - bz \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & a & 0 \\ 0 & 1 & b & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

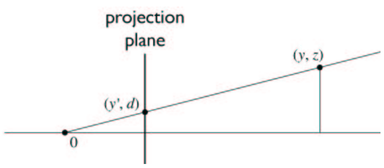
Parallel projection: oblique



to implement oblique, rewrite in oblique basis, then toss out z:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} x - az \\ y - bz \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -a & 0 \\ 0 & 1 & -b & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Perspective projection



similar triangles:

$$\frac{y'}{d} = \frac{y}{z}$$
$$y' = dy/z$$

Homogeneous coordinates revisited

- Perspective requires division
  - that is not part of affine transformations
  - in affine, parallel lines stay parallel
    - therefore not vanishing point
    - therefore no rays converging on viewpoint
- "True" purpose of homogeneous coords: projection

Homogeneous coordinates revisited

- Introduced *w* = 1coordinate as a placeholder

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

  - used as a convenience for unifying translation with linear
- Can also allow arbitrary *w*

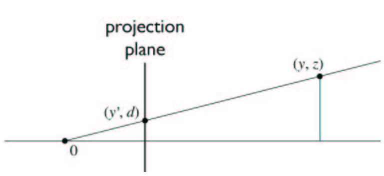
$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \sim \begin{bmatrix} wx \\ wy \\ wz \\ w \end{bmatrix}$$

Implications of *w*

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \sim \begin{bmatrix} wx \\ wy \\ wz \\ w \end{bmatrix}$$

- All scalar multiples of a 4-vector are equivalent
- When *w* is not zero, can divide by *w*
  - therefore these points represent "normal" affine points
- When *w* is zero, it's a point at infinity
  - this is the point where parallel lines intersect
  - can also think of it as the vanishing point
- Digression on projective space

Perspective projection



to implement perspective, just move z to w:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} dx/z \\ dy/z \\ 1 \end{bmatrix} \sim \begin{bmatrix} x \\ y \\ z/d \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

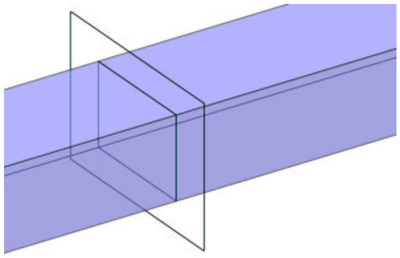
View volumes

- The volume of space that ends up in the image

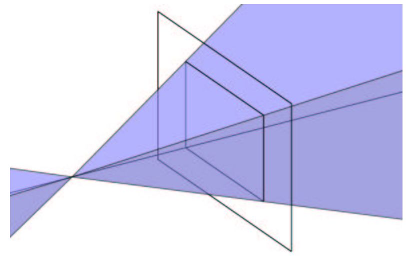
$$V = \{\mathbf{p} \mid P\mathbf{p} \in R\}$$

  - *P* is the projection matrix; *R* is the image rectangle

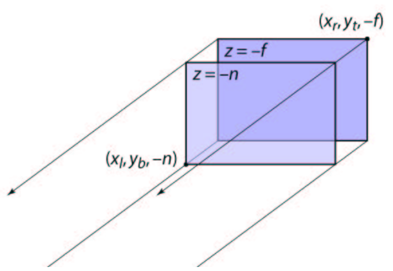
View volume: orthographic



View volume: perspective



View volume: orthographic



View volume: perspective