CS 417 Prelim 1

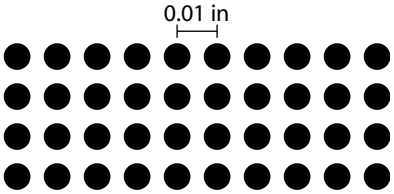
Tuesday, February 25, 2003

Problem 1: Raster images (15 pts)

1. (10 pts) Order the following three images by how much memory they occupy:
   A. a 2048 by 2048 binary image
   B. a 1024 by 1024 grayscale image with 256 gray levels
   C. a 512 by 512 color image with 256 levels in each color component
2. (5 pts) Which image would be most appropriate for direct output to each of the following devices:
   i. An RGB LCD
   ii. A laser printer
   iii. A black-and-white CRT

Problem 2: Moiré patterns (25 pts)

Suppose we have a book that contains a large area of uniform gray that was printed with a halftone screen of 100 lines per inch. That is, the paper is printed with a square grid of round dots, spaced 0.01 inch apart in x and y:



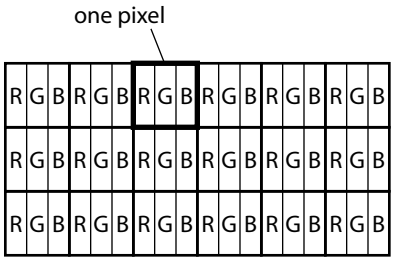
(Note that a gray halftone is normally printed with a grid rotated 45° from the one shown here, but we'll use this one for this problem to keep things simple.)

1. (20 pts) Suppose we scan this image at 101 pixels per inch to produce an image of 300 x 300 pixels. If we stand back and look at the image as a whole, what will the resulting image look like? What are the main features you will see? Draw a sketch to illustrate. Assume that the scanner has very sharp optics so that each measurement is essentially a point sample, and assume that the pixel in the lower left corner of the image is positioned at the center of one of the halftone dots.

2. (5 pts) How will your answer change if we increase the scanning resolution to 102 pixels per inch?

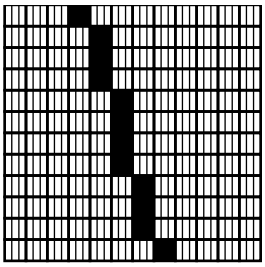
Problem 3: Rasterization (30 pts)

This problem is about drawing nearly vertical white lines against a black background on a color LCD display. The display has square pixels, each of which is composed of red, green, and blue sub-pixels. It looks like this in closeup:

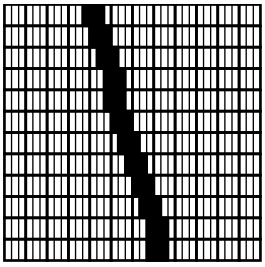


When a pixel is set to the value (r, g, b), the red sub-pixel, labeled R in the drawing, lights up with intensity r, the green sub-pixel lights up with intensity g, and the blue sub-pixel lights up with intensity b.

If we draw on this display in the normal way, by turning some pixels on white and leaving the rest off, the lines look like this:[1]



However, the display is perfectly capable of displaying this image instead:



Each row still has a triple of adjacent sub-pixels turned on, so the line still looks white overall, even though the colors that are stored in the pixels are not white.

Here is pseudocode for one case of Bresenham's algorithm to draw a normal line from the point (x_0, y_0) to the point (x_1, y_1):

[1] The pixels that are filled in in the drawing are the ones that are turned on, not the ones that are black.

```
drawLine(int x0, int y0, int x1, int y1)
   float m = (x1 - x0) / (y1 - y0)
   int x = x0
   int y = y0
   float d = m
   output(x0,  y0, WHITE)
   while y < y1
      if d > 0.5
         x += 1
         d -= 1
      y += 1
      d += m
      output(x, y, WHITE)
```

This code works in the case where $x_1 > x_0$ and $y_1 > y_0$ and $x_1 - x_0 < y_1 - y_0$.

1. (30 pts) Modify this pseudocode to draw a smoother line as illustrated above. Your code should set pixels to `RED`, `BLUE`, `YELLOW` (red + green), and `CYAN` (green + blue) as well as `WHITE`. In addition to the assumptions made by the given code, you may assume that $3(x_1 - x_0) < y_1 - y_0$.

Problem 4: 2D Geometry (30 pts)

1. (20 pts) Write down the 3x3 homogeneous matrices for the following affine transformations. Your answer should be an array of 9 numbers in each case, but the numbers may be something like "$3\sqrt{2}$". Show your work, if any, for each.

   (a) a translation by 3 along x and 2 along y

   (b) a counterclockwise rotation by 45° about the origin

   (c) a scale by 1.4 along the x axis

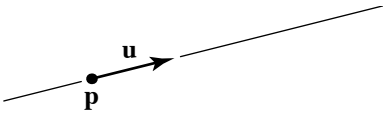
   (d) a counterclockwise rotation by 30° about the origin followed by a translation by 4 along the x axis

   (e) a translation by 4 along the x axis followed by a counterclockwise rotation by 30° about the origin

   (f) a counterclockwise rotation by 30° about the point $(4, 0)$

An infinite line is represented parametrically as

$$\{\mathbf{p} + t\mathbf{u} \,|\, t \in I\!R\}$$

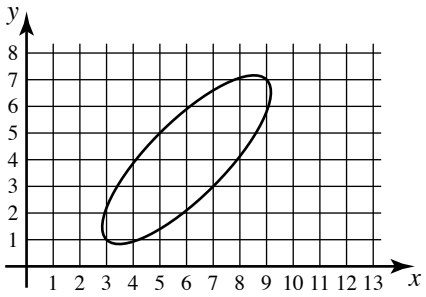


2. (3 pts) Let $T(\mathbf{x}) = M\mathbf{x} + \mathbf{w}$. What is the parametric representation of $T(L)$?

The unit circle may be represented parametrically as

$$\{[\cos t, \sin t]^T \,|\, t \in [0, 2\pi]\}$$

3. (7 pts) Give a parametric representation for the ellipse shown here:



   Make use of affine transformations to keep your expression simple. It is fine to write your answer as a product of several matrices.