CS 4120
Introduction to Compilers

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Lecture 19: Live Variable Analysis

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Problem

- Abstract assembly contains arbitrarily many registers t_i
- Want to replace all such nodes with register nodes
- Local variables allocated to TEMP's too
- Only 9–15 usable registers: need to allocate multiple t_i to each register
- For each statement, need to know which variables are *live* to reuse registers

Using scope

- Observation: temporaries, variables have bounded scope in program
- Simple idea: use information about program scope to decide which variables are live
- Problem: overestimates liveness

```
{   int b = a + 2;        ← b is live
    int c = b*b;          ← c is live, b is not
    int d = c + 1;        ← what is live here?
    return d; }
```

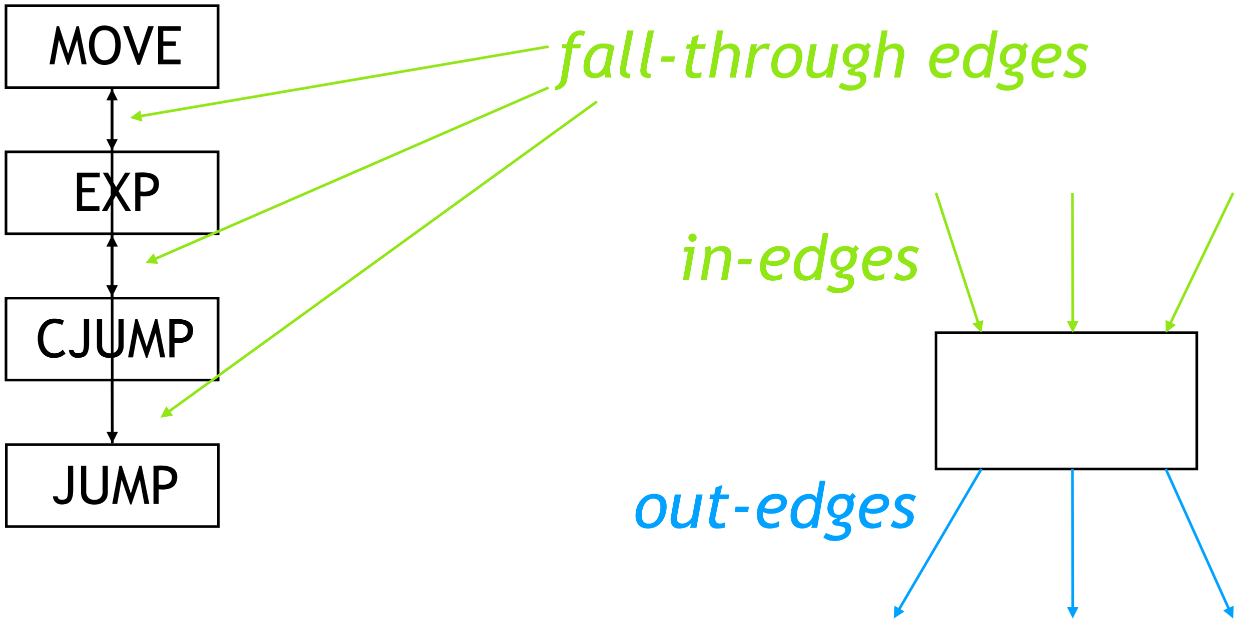
Live variable analysis

- Goal: for each statement, identify which temporaries are live
- Analysis will be *conservative* (may over-estimate liveness, will never under-estimate)

But more *precise* than simple scope analysis (will estimate fewer live temporaries)

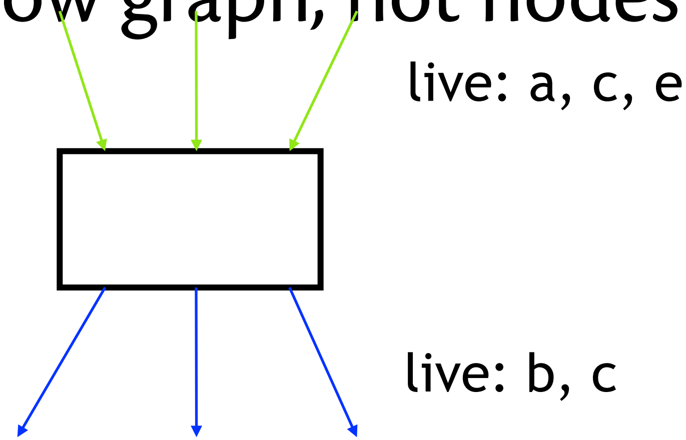
Control Flow Graph

- Canonical IR forms *control flow graph* (*CFG*) : statements are nodes; jumps, fall-throughs are edges



Liveness

- Liveness is associated with *edges* of control flow graph, not nodes (statements)



- Same register can be used for different temporaries manipulated by one stmt

Example

a = b + 1

⇩

MOVE(TEMP(ta), TEMP(tb) + 1)

⇩

mov tb, ta

add $1, ta

Live: tb
mov tb, ta
add $1, ta
Live: ta (maybe)

Register allocation: ta ⇒ rax, tb ⇒ rax

~~mov rax, rax~~

add $1, rax

Use/Def

- Every statement *uses* some set of variables (reads from them) and *defines* some set of variables (writes to them)
  - For statement s define:
    - $use[s]$: set of variables used by s
    - $def[s]$: set of variables defined by s
  - Example:

a = b + c	*use* = b,c	*def* = a
a = a + 1	*use* = a	*def* = a

Liveness

Variable v is *live* on edge e if:

There is

- a node n in the CFG that uses it *and*
- a directed path from e to n passing through no *def*

How to compute efficiently?

How to use?

Simple algorithm: Backtracing

"variable v is *live* on edge e if there is a node n in CFG that uses it *and* a directed path from e to n passing through no *def* for v"

(Slow) algorithm: Try all paths from each *use* of a variable, tracing *backward* in the control flow graph until a *def* node or previously visited node is reached. Mark variable live on each edge traversed.

Dataflow Analysis

- *Idea*: compute liveness for all variables simultaneously
  - Approach: define *equations* that must be satisfied by any liveness determination
  - Solve equations by iteratively converging on solution
  - Instance of general technique for computing program properties: *dataflow analysis*

Dataflow values

$use[n]$: set of variables used by n

$def[n]$: set of variables defined by n

$in[n]$: variables live on entry to n

$out[n]$: variables live on exit from n

Clearly: $in[n] \supseteq use[n]$

What other constraints are there?

Dataflow constraints

$in[n] \supseteq use[n]$

- A variable must be live on entry to n if it is used by the statement itself

$in[n] \supseteq out[n] - def[n]$

- If a variable is live on output and the statement does not define it, it must be live on input too

$out[n] \supseteq in[n'] \quad \text{if} \quad n' \in succ[n]$

- if live on input to n', must be live on output from n

Iterative dataflow analysis

- Initial assignment to $in[n]$, $out[n]$ is empty set Ø : will not satisfy constraints

$$in[n] \supseteq use[n]$$

$$in[n] \supseteq out[n] - def\,[n]$$

$$out[n] \supseteq in[n'\,] \quad \text{if} \quad n' \in succ\,[n]$$

- Idea: iteratively re-compute in[n], out[n] when forced to by constraints. Live variable sets will increase monotonically.
- Dataflow equations:

$$in'[n] = use[n] \cup (out[n] - def\,[n])$$

$$out'[n] = \bigcup_{n' \in succ[n]} in[n']$$

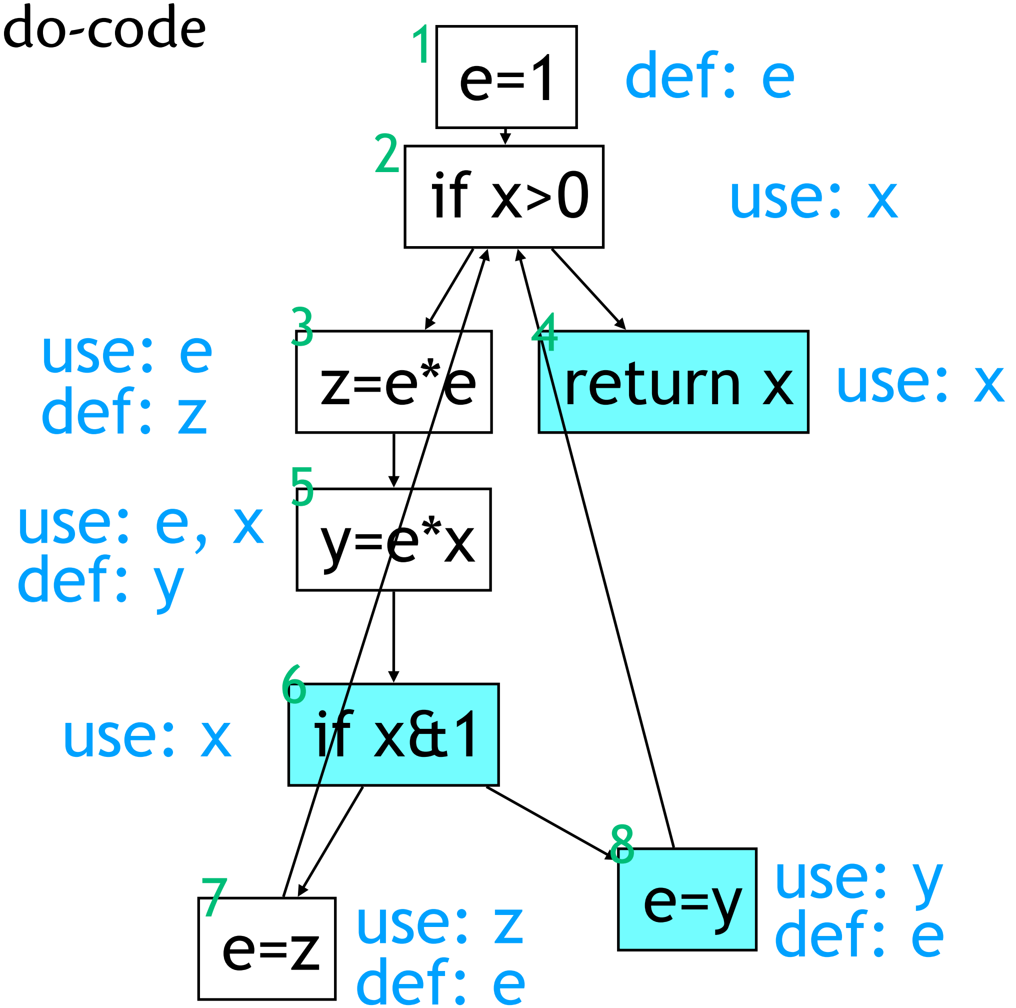
Complete algorithm

```
for all n, in[n] = out[n] = Ø
repeat until no change
    for all n
            out[n] = ∪n' ∈ succ[n] in[n']
            in[n] = use[n] ∪ (out[n] - def[n])
    end
end
```

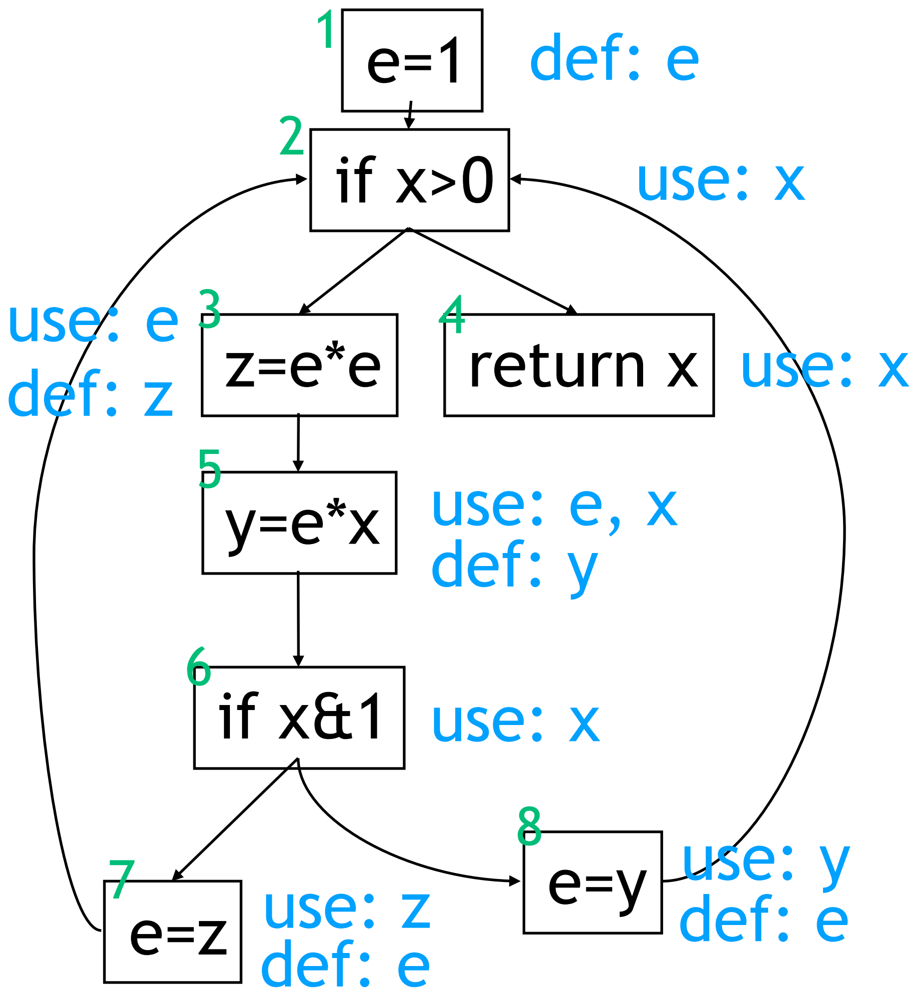
- Finds *fixed point* of in, out equations
- But: does extra work recomputing in, out values when no change can happen

Example

- For simplicity: pseudo-code



Example



2: in={x}
3: in={e}
4: in={x}
5: in={e,x}
6: in={x}
7: out={x}, in={x,z}
8: out={x}, in={x,y}
1: out={x}, in={x}
2: out={e,x}, in={e,x}
3: out={e,x}, in={e,x}
5: out={x}, in={e,x}
6: out={x,y,z}, in={x,y,z}
7: out={e,x}, in={x,z}
8: out={e,x}, in={x,y}
1: out={e,x}, in={x}
5: out={x,y,z}, in={e,x,z}
3: out={e,x,z}, in={e,x}
all equations satisfied

Faster algorithm

- Information only propagates between nodes because of this equation:

$$out[n] = \bigcup_{n' \in succ[n]} in[n']$$

- Node is updated from its successors
  - If successors haven't changed, no need to apply equation for node
  - Should start with nodes at "end" and work backward

Worklist algorithm

- **Idea:** keep track of nodes that might need to be updated in *worklist* : FIFO queue

```
for all n, in[n] = out[n] = Ø
w = { set of all nodes }
repeat until w empty
     n = w.pop( )
     out[n] = ∪_{n' ∈ succ [n]} in[n']
     in[n] = use[n] ∪ (out[n] – def [n])
     if change to in[n],
          for all predecessors m of n, w.push(m)
end
```

Running time

- out[n] can change at most V times where V is the number of variables.
- How many times is a node pushed onto the worklist?
  - once at beginning
  - at most once for each time successor nodes are updated; at most 2 successors
- Total pushes per node: at most 2V+1.

$\Rightarrow$ O(NV) total node updates