CS412/CS413

Introduction to Compilers
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Lecture 14: Static Semantics
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Static Semantics

- Static semantics defines types for all legal ASTs in the language
- Is to type-checking:
  - As grammar is to syntax analysis
  - As regular expression is to lexical analysis

Type Judgments

- Static semantics is a formal notation that describes type judgments:

  |– E : T

  meaning:

  "E is a well-typed expression of type T"

- Type judgment examples:

  |– 2 : int |– 2 * (3 + 4) : int

  |– true : bool |– "Hello" : string

Type Judgments for Statements

- Statements may be expressions (i.e., represent values)
- Use type judgments for statements:

  |– **if** (b) **then** 2 **else** 3 : int
  |– x = **false** : bool
  |– b = **true**; y = 2 : int

- For statements that are not expressions: use a special unit type (empty type):

  |– S : unit

  means "S is a well-typed statement with no result type"

Deriving a Judgment

- Consider the judgment:

  |– **if** (b) **then** 2 **else** 3 : int

- What do we need to decide that this is a well-typed expression of type int?
- b must be a bool (|– b: bool)
- 2 must be an int (|– 2: int)
- 3 must be an int (|– 3: int)

Hypothetical Type Judgments

- Type judgment notation: A |– E : T
  means "In the type context A the expression E is a well-typed expression with the type T "
- Type context is a set of type bindings id : T
  (i.e., type context = symbol table)

  b: bool, x: int |– b : bool
  b: bool, x: int |– **if** (b) **then** 2 **else** x : int
  |– 2 + 2 : int

Deriving a Judgment

- To show:

  b: bool, x: int |– **if** (b) **then** 2 **else** x : int

- Need to show:

  b: bool, x: int |– b : bool
  b: bool, x: int |– 2 : int
  b: bool, x: int |– x : int

General Rule

- For any environment A, expression E, statements S_1 and S_2, the judgment

  A |– **if** (E) **then** S_1 **else** S_2 : T

  is valid if:

  A |– E : bool
  A |– S_1 : T
  A |– S_2 : T

Inference Rules

Premises

$$\frac{A \vdash E: \text{bool} \quad A \vdash S_1: T \quad A \vdash S_2: T}{A \vdash \textbf{if } (E) \textbf{ then } S_1 \textbf{ else } S_2 : T} \text{ (if-rule)}$$

Conclusion

- Holds for any choice of E, S_1, S_2, T

Why Inference Rules?

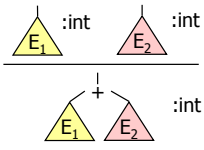
- Inference rules: compact, precise language for specifying static semantics (can specify languages in ~20 pages vs. 100's of pages of Java Language Specification)
- Inference rules correspond directly to recursive AST traversal that implements them
- Type checking is attempt to prove that type judgments A |– E : T are derivable

Meaning of Inference Rule

- Inference rule says:
  given that antecedent judgments are derivable
  - with some substitution for A, E_1, E_2

  then, consequent judgment is derivable
  - with a consistent substitution

$$\frac{A \vdash E_1 : \text{int} \quad A \vdash E_2 : \text{int}}{A \vdash E_1 + E_2 : \text{int}} (+)$$



Proof Tree

- Expression is well-typed if there exists a type derivation for a type judgment
- Type derivation is a proof tree
- Example: if A1 = b: bool, x: int, then:

$$\dfrac{\dfrac{\dfrac{}{A1 \vdash b: \text{bool}}}{A1 \vdash !b: \text{bool}} \quad \dfrac{\dfrac{}{A1 \vdash 2: \text{int}} \quad \dfrac{}{A1 \vdash 3: \text{int}}}{A1 \vdash 2+3 : \text{int}} \quad \dfrac{}{A1 \vdash x : \text{int}}}{A1 \vdash \textbf{if } (!b) \textbf{ then } 2+3 \textbf{ else } x : \text{int}}$$

More about Inference Rules

- No premises = axiom

$$\frac{}{A \vdash \textbf{true} : \text{bool}}$$

- A goal judgment may be proved in more than one way

$$\frac{A \vdash E_1 : \text{float} \quad A \vdash E_2 : \text{float}}{A \vdash E_1 + E_2 : \text{float}} \qquad \frac{A \vdash E_1 : \text{float} \quad A \vdash E_2 : \text{int}}{A \vdash E_1 + E_2 : \text{float}}$$

- No need to search for rules to apply -- they correspond to nodes in the AST

While Statements

- Rule for while statements:

$$\frac{A \vdash E : \text{bool} \quad A \vdash S : T}{A \vdash \textbf{while}\ (E)\ S : \text{unit}}\ \text{(while)}$$

- Why unit type?

If Statements

- If statement as an expression (e.g., in ML): its value is the value of the branch that is executed

$$\frac{A \vdash E : \text{bool} \quad A \vdash S_1 : T \quad A \vdash S_2 : T}{A \vdash \textbf{if}\ (E)\ \textbf{then}\ S_1\ \textbf{else}\ S_2 : T}\ \text{(if-then-else)}$$

- If no else clause:

$$\frac{A \vdash E : \text{bool} \quad A \vdash S : T}{A \vdash \textbf{if}\ (E)\ S : ?}\ \text{(if-then)}$$

Assignment Statements

$$\frac{A, id : T \vdash E : T}{A, id : T \vdash id = E : T}\ \text{(variable-assign)}$$

$$\frac{A \vdash E_3 : T \quad A \vdash E_2 : \text{int} \quad A \vdash E_1 : \text{array}[T]}{A \vdash E_1[E_2] = E_3 : T}\ \text{(array-assign)}$$

Sequence Statements

- Rule: A sequence of statements is well-typed if the first statement is well-typed, and the remaining are well-typed too:

$$\frac{A \vdash S_1 : T_1 \quad A \vdash (S_2 ; \ldots ; S_n) : T_n}{A \vdash (S_1 ; S_2 ; \ldots ; S_n) : T_n}\ \text{(sequence)}$$

Declarations

- What about variable declarations?
- Declarations add entries to the environment (in the symbol table)

$$\frac{A \vdash id : T\ [\ = E\ ] : T_1 \quad A, id : T \vdash (S_2 ; \ldots ; S_n) : T_n}{A \vdash (id : T\ [\ = E\ ]; S_2 ; \ldots ; S_n) : T_n}\ \text{(declaration)}$$

= unit if no E

Function Calls

- If expression E is a function value, it has a type $T_1 \times T_2 \times \ldots \times T_n \rightarrow T_r$
- T_i are argument types; T_r is return type
- How to type-check function call $E(E_1,\ldots,E_n)$?

$$\frac{A \vdash E : T_1 \times T_2 \times \ldots \times T_n \rightarrow T_r \quad A \vdash E_i : T_i \ \ (i \in 1..n)}{A \vdash E(E_1,\ldots,E_n) : T_r} \ \text{(function-call)}$$

Function Declarations

- Consider a function declaration of the form

  T_r fun $(T_1\ a_1,\ldots,\ T_n\ a_n)$ { **return** E; }

- Type of function body S must match declared return type of function, i.e., $E : T_r$
- … but in what type context?

Add Arguments to Environment!

- Let A be the context surrounding the function declaration. Then the function declaration

  T_r fun $(T_1\ a_1,\ldots,\ T_n\ a_n)$ { **return** E; }

  is well-formed if

  $A, a_1 : T_1, \ldots, a_n : T_n \vdash E : T_r$

- …but what about recursion?

  Need: fun: $T_1 \times T_2 \times \ldots \times T_n \rightarrow T_r \in A$

Recursive Function Example

- Factorial:

```
int fact(int x)  {
    if (x==0) return 1;
    else return x * fact(x - 1);
}
```

- Prove: $A \vdash$ x * fact(x-1) : int

  Where: A = { fact: int→int, x : int }

Mutual Recursion

- Example:

```
int f(int x) { return g(x) + 1; }
int g(int x) { return f(x) – 1; }
```

- Need environment containing at least

  f: int → int, g: int → int

  when checking both f and g

- Two-pass approach:
  - Scan top level of AST picking up all function signatures and creating an environment binding all global identifiers
  - Type-check each function individually using this global environment

How to Check Return?

$$\frac{A \vdash E : T}{A \vdash \textbf{return}\ E : \text{unit}} \ \text{(return1)}$$

- A return statement produces no value for its containing context to use
- Does not return control to containing context
- Suppose we use type unit…
- …then how to make sure the return type of the current function is T ?

Put Return in the Symbol Table

- Add a special entry { return_fun : T } when we start checking the function "fun", look up this entry when we hit a return statement.
- To check T_r fun (T_1 a_1,..., T_n a_n) { **return** S; } in environment A, need to check:

$$A, a_1 : T_1, \ldots, a_n : T_n, \text{return\_fun} : T_r \vdash S : T_r$$

$$\frac{A \vdash E : T \qquad \text{return\_fun} : T \in A}{A \vdash \textbf{return } E : \text{unit}} \text{ (return)}$$

Static Semantics Summary

- Static semantics = formal specification of type-checking rules
- Concise form of static semantics: typing rules expressed as inference rules
- Expression and statements are well-formed (or well-typed) if a typing derivation (proof tree) can be constructed using the inference rules