CS412/CS413

Introduction to Compilers
Tim Teitelbaum

Lecture 9: LR Parsing
February 11, 2005

LR(k) Grammars

- LR(k) = Left-to-right scanning, Right-most derivation, k look-ahead characters
- Main cases: LR(0), LR(1), and some variations (SLR and LALR(1))
- Parsers for LR(0) Grammars:
  - Determine the actions without any lookahead symbol
  - will help us understand shift-reduce parsing

Building LR(0) Parsing Tables

- To build the parsing table:
  - Define states of the parser
  - Build a DFA to describe the transitions between states
  - Use the DFA to build the parsing table

Viable Prefix

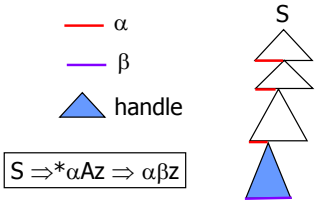
- γ is a viable prefix for G iff there is some derivation

  $S \Rightarrow^* \alpha Az \Rightarrow \alpha\beta z$

  where γ is a prefix of $\alpha\beta$

Viable Prefix (Informally)

- γ is a viable prefix for G if it is a prefix of a sentential form derived from S that does not extend past the end of the handle of the sentential form.



LR(0) Items

- An LR(0) item for G is a triple $\langle A, \beta_1, \beta_2 \rangle$ such that $A \rightarrow \beta_1\beta_2$ is a production of G. The item $\langle A, \beta_1, \beta_2 \rangle$ will always be denoted by $[A \rightarrow \beta_1 . \beta_2]$

Validity of LR(0) Items

- The item $[A \rightarrow \beta_1 . \beta_2]$ is valid for viable prefix $\alpha\beta_1$ iff $S \Rightarrow^* \alpha Az \Rightarrow \alpha\beta_1\beta_2 z$
- Note:
  - β_1 may be ε
  - β_2 may be ε
- For any viable prefix α, we let $V(\alpha)$ denote the set of LR(0) items that are valid for α.

Sets of Valid Items

- Observations
  - There are only finitely many distinct LR(0) items for a given G.
  - Thus, there are only finitely many sets of LR(0) items for G.
- Sets of valid items for various viable prefixes will serve as the states of a DFA.

Relation $\downarrow$

- The relation $\downarrow$ on LR(0) items is defined by $I \downarrow I'$ iff $\exists$ A, B, β_1, β_2, β_3 such that
  $I = [A \rightarrow \beta_1 . B\beta_3]$
  $I' = [B \rightarrow . \beta_2]$
- Lemma. Let I, I′ be as above. If $I \in V(\alpha\beta_1)$ and $I \downarrow I'$, then $I' \in V(\alpha\beta_1)$.
  - $I \in V(\alpha\beta_1)$ implies $S \Rightarrow^* \alpha Az \Rightarrow \alpha\beta_1 B\beta_3 z$
  - Assuming G has no useless productions, $\exists y$ such that $\beta_3 \Rightarrow^* y$
  - Thus, $S \Rightarrow^* \alpha Az \Rightarrow \alpha\beta_1 B\beta_3 z \Rightarrow^* \alpha\beta_1 Byz \Rightarrow \alpha\beta_1\beta_2 yz$
  - Thus, I′ (i.e., $[B \rightarrow . \beta_2]$) $\in V(\alpha\beta_1)$

Relation $\rightarrow_x$

- For any $X \in (V \cup \Sigma)$, the relation $\rightarrow_x$ is defined by $I \rightarrow_x I'$ iff $\exists$ A, β_1, β_3 such that
  $I = [A \rightarrow \beta_1 . X\beta_3]$
  $I' = [A \rightarrow \beta_1 X . \beta_3]$
- Lemma. Let I, I′ be as above. If $I \in V(\alpha\beta_1)$ then $I' \in V(\alpha\beta_1 X)$.
  - $I = [A \rightarrow \beta_1 . X\beta_3] \in V(\alpha\beta_1)$ implies
    $S \Rightarrow^* \alpha Az \Rightarrow \alpha\beta_1 X\beta_3 z$
    which by definition means I' (= $[A \rightarrow \beta_1 X . \beta_3]$) $\in V(\alpha\beta_1 X)$

Technical Details

- Start symbol never appears on RHS
  - It is convenient if the start symbol never appears on the RHS of any production.
  - Given $G = \langle V, \Sigma, S, \rightarrow \rangle$, let $S' \notin V$ and
    $G' = \langle V, \Sigma, S', \rightarrow \cup \{S' \rightarrow S\} \rangle$
  - We assume that the grammars we work with have the form of G′.
- If S is a set and R is a relation, then
  $SR = \{y \mid x \in S \text{ and } \langle x, y \rangle \in R\}$
  SR is called S mapped by R, i.e,

V(ε), the base case

- Let S′ be the start symbol of G. Then
  - $V(\varepsilon) = \{ [S' \rightarrow . S] \}\downarrow^*$

  (i.e., the above "initial" items of G mapped by the reflexive transitive closure of the $\downarrow$ relation.)
- If Q is a set of items, we call $Q\downarrow^*$ the closure(Q).

V(αX), the inductive case

- For any α and X, $V(\alpha X) = V(\alpha)\rightarrow_x\downarrow^*$
- For any set Q of items, we call $Q\rightarrow_x\downarrow^*$ the X-successor of Q, or Goto(Q,X).

Cannonical LR(0) Machine

- States: Sets of valid items
- Transition function: Goto, as defined above.
- Algorithm: To compute all sets of valid items

  STATES := V(ε)
  while ∃ Q ∈ STATES, X∈(V∪Σ) such that
  Goto(Q,X) ∉ STATES
  do STATES := STATES ∪ { Goto(Q,X) }

- Clearly, this terminates, as STATES is bounded above by the Powerset(LR(0) items)

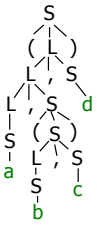
LR(0) Grammar

- Nested lists:

  S → (L) | id
  L → S | L,S

- Sample strings
  - (a,b,c)
  - ((a,b),(c,d),(e,f))
  - (a,(b,c,d),((f,g)))

Parse tree for (a, (b,c), d)



Start State

Grammar
S → (L) | id
L → S | L, S

- Start state
  - $V(\varepsilon) = \{ [S'\rightarrow .S] \}\downarrow^*$
    $= \{ [S'\rightarrow .S] [S\rightarrow .(L)], [S\rightarrow .id] \}$
- Closure of a parser state Q:
  - Start with Closure(Q) = Q
  - Then for each item in Q:

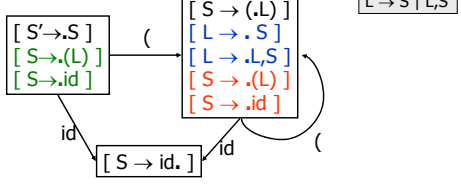
    $A \rightarrow \alpha.B\beta$

    add the items for all the productions $B \rightarrow \gamma$ to the closure of Q:

    $B \rightarrow .\ \gamma$

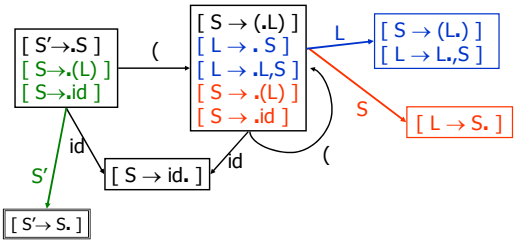
Goto: Terminal Symbols

Grammar
S → (L) | id
L → S | L,S



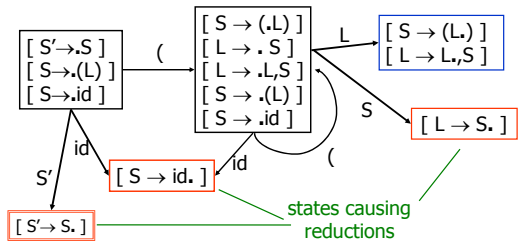
In new state, include all items that have appropriate input symbol just after dot, advance dot in those items, and take closure.

Goto: Nonterminal Symbols

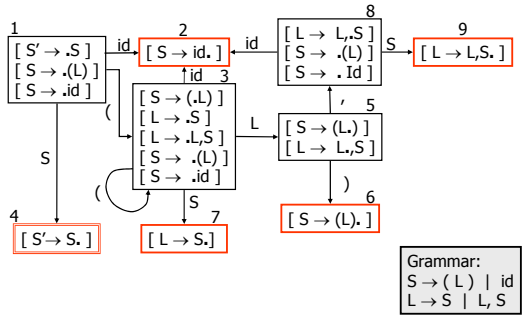


(same algorithm for transitions on nonterminals)

Reduce States



Full LR(0) Machine



Parsing Example: ((a),b)

Grammar:
S → (L) | id
L → S | L, S

derivation	stack	input	action
((a),b) ⇐	1	((a),b)	shift, goto 3
((a),b) ⇐	13	(a),b)	shift, goto 3
((a),b) ⇐	133	a),b)	shift, goto 2
((a),b) ⇐	1332	),b)	reduce S→id
((S),b) ⇐	1337	),b)	reduce L→S
((L),b) ⇐	1335	),b)	shift, goto 6
((L),b) ⇐	13356	,b)	reduce S→(L)
(S,b) ⇐	137	,b)	reduce L→S
(L,b) ⇐	135	,b)	shift, goto 8
(L,b) ⇐	1358	b)	shift, goto 9
(L,b) ⇐	13582	)	reduce S→id
(L,S) ⇐	13589	)	reduce L→L , S
(L) ⇐	135	)	shift, goto 6
(L) ⇐	1356		reduce S→(L)
S	14		done

Reductions

- On reducing B→β with stack $\alpha\beta_2$:
  - pop $|\beta|$ states off stack
  - This reveals topmost state Q, which contains an item $[A \rightarrow \beta_1.B\beta_3]$
  - push state Goto(Q,B) onto the stack

LR(0) Parsing Table

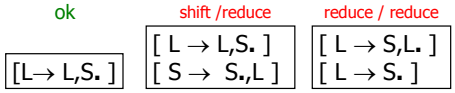
	(	)	id	,	ε	S	L
1	s3		s2			g4	
2	S→id	S→id	S→id	S→id	S→id		
3	s3		s2			g7	g5
4					accept		
5		s6		s8			
6	S→(L)	S→(L)	S→(L)	S→(L)	S→(L)		
7	L→S	L→S	L→S	L→S	L→S		
8	s3		s2			g9	
9	L→L,S	L→L,S	L→L,S	L→L,S	L→L,S		

LR(0) Summary

- LR(0) parsing recipe:
  Start with an LR(0) grammar
  Compute LR(0) states and build DFA:
  Build the LR(0) parsing table from the DFA

LR(0) Limitations

- An LR(0) machine only works if states with reduce actions have a single reduce action -- in those states, always reduce ignoring lookahead
- With more complex grammar, construction gives states with shift/reduce or reduce/reduce conflicts
- Need to use look-ahead to choose

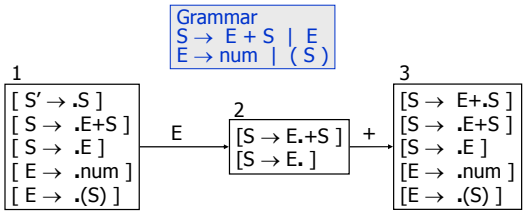


A Non-LR(0) Grammar

- Grammar for addition of numbers:
  S → S + E | E
  E → num | (S)
- Left-associative is LR(0)
- Right-associative version is not LR(0)
  S → E + S | E
  E → num | (S)

LR(0) Parsing Table

Grammar
S → E + S | E
E → num | (S)



What to do in state 2?

	+	ε	E
1			2
2	s3/S→E	S→E	

SLR(k)

- Use the LR(0) machine states as rows of table
- Let Q be a state and u be a lookahead string
  - Action(Q,u) = shift Goto(Q,b)
    if Q contains an item of the form $[A \rightarrow \beta_1.b\beta_3]$, with $u \in FIRST_k(b\beta_3\ FOLLOW_k(A))$
  - Action(Q,u) = accept
    if Q = { [S'→S] } and u=ε
  - Action(Q,u) = reduce i
    if Q contains the item $[A \rightarrow \beta.]$, where $A \rightarrow \beta$ is the ith production of G and $u \in FOLLOW_k(A)$
  - Action(Q,u) = error otherwise
- G is SLR(k) iff the Action function given above is single-valued for all Q and u, i.e, there are no shift-reduce or reduce-reduce conflicts.

Next Time

- Learn about other kinds of LR parsing:
  - SLR = improved LR(0)
  - LR(1) = 1 character lookahead
  - LALR(1) = Look-Ahead LR(1)
- Basic ideas are the same as for LR(0)
  - Parser states with LR items
  - DFA with transitions between parser states
  - Parser table with shift/reduce/goto actions