

CS412/CS413

Introduction to Compilers Tim Teitelbaum

Lecture 8: LR parsing
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Bottom-up Parsing

- A more powerful parsing technology
- LR grammars -- more expressive than LL
 - Construct **right-most derivation** of program
 - Left-recursive grammars, virtually all programming languages
 - Easier to express programming language syntax
- Shift-reduce parsers
 - Parsers for LR grammars
 - Automatic parser generators (*e.g.*, yacc, CUP)

Bottom-up Parsing

- Right-most derivation -- backward
 - Start with the tokens; end with the start symbol

$(1+2+(3+4))+5 \Leftarrow$
 $(E+2+(3+4))+5 \Leftarrow$
 $(S+2+(3+4))+5 \Leftarrow$
 $(S+E+(3+4))+5 \Leftarrow$
 $(S+(3+4))+5 \Leftarrow$
 $(S+(E+4))+5 \Leftarrow$
 $(S+(S+4))+5 \Leftarrow$
 $(S+(S+E))+5 \Leftarrow$
 $(S+(S)))+5 \Leftarrow$
 $(S+E)+5 \Leftarrow$
 $(S)+5 \Leftarrow$
 $E+5 \Leftarrow$
 $S+5 \Leftarrow$
 $S+E \Leftarrow$
 S

$S \rightarrow S + E \mid E$
 $E \rightarrow \text{num} \mid (S)$

Progress of Bottom-up Parsing

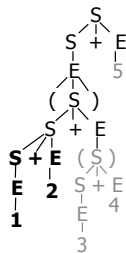
$(1+2+(3+4))+5 \Leftarrow$
 $(E+2+(3+4))+5 \Leftarrow$
 $(S+2+(3+4))+5 \Leftarrow$
 $(S+E+(3+4))+5 \Leftarrow$
 $(S+(3+4))+5 \Leftarrow$
 $(S+(E+4))+5 \Leftarrow$
 $(S+(S+4))+5 \Leftarrow$
 $(S+(S+E))+5 \Leftarrow$
 $(S+(S)))+5 \Leftarrow$
 $(S+E)+5 \Leftarrow$
 $(S)+5 \Leftarrow$
 $E+5 \Leftarrow$
 $S+5 \Leftarrow$
 $S+E \Leftarrow$
 S

↑ right-most derivation

Bottom-up Parsing

- $(1+2+(3+4))+5 \Leftarrow$
 $(E+2+(3+4))+5 \Leftarrow$
 $(S+2+(3+4))+5 \Leftarrow$
 $(S+E+(3+4))+5 \dots$

$S \rightarrow S + E \mid E$
 $E \rightarrow \text{num} \mid (S)$



- Advantage of bottom-up parsing: can postpone the selection of productions until more of the input is scanned

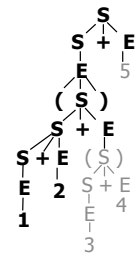
Top-down Parsing

$(1+2+(3+4))+5$

$S \rightarrow S + E \mid E$
 $E \rightarrow \text{num} \mid (S)$

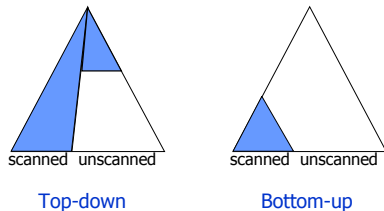
$S \Rightarrow S+E \Rightarrow E+E \Rightarrow (S)+E \Rightarrow (S+E)+E$
 $\Rightarrow (S+E+E)+E \Rightarrow (E+E+E)+E$
 $\Rightarrow (1+E+E)+E \Rightarrow (1+2+E)+E \dots$

- In left-most derivation, entire tree above a token (2) has been expanded when encountered



Top-down vs. Bottom-up

Bottom-up: Don't need to figure out as much of the parse tree for a given amount of input



Shift-reduce Parsing

- **Parsing actions:** a sequence of **shift** and **reduce** operations
- **Parser state:** a stack of terminals and non-terminals (grows to the right)
- Current derivation step = always stack+input

Derivation step	stack	unconsumed input
$(1+2+(3+4))+5 \Leftarrow$	($(1+2+(3+4))+5$
	(1	$1+2+(3+4))+5$
	(1	$+2+(3+4))+5$
$(E+2+(3+4))+5 \Leftarrow$	(E	$+2+(3+4))+5$
$(S+2+(3+4))+5 \Leftarrow$	(S	$+2+(3+4))+5$
	(S+	$2+(3+4))+5$
	(S+2	$+(3+4))+5$
$(S+E+(3+4))+5 \Leftarrow$	(S+E	$+(3+4))+5$

Shift-reduce Parsing

- Parsing is a sequence of shifts and reduces
- **Shift:** move look-ahead token to stack

stack	input	action
($1+2+(3+4))+5$	shift 1
(1	$+2+(3+4))+5$	

- **Reduce:** Replace symbols β from top of stack with non-terminal symbol X , corresponding to production $A \rightarrow \beta$ (pop β , push A)

stack	input	action
$(S+E$	$+(3+4))+5$	reduce $S \rightarrow S+E$
(S	$+(3+4))+5$	

Shift-reduce Parsing

$S \rightarrow S + E \mid E$
 $E \rightarrow \text{num} \mid (S)$

derivation	stack	input stream	action
$(1+2+(3+4))+5 \Leftarrow$	($(1+2+(3+4))+5$	shift
$(1+2+(3+4))+5 \Leftarrow$	(1	$1+2+(3+4))+5$	shift
$(1+2+(3+4))+5 \Leftarrow$	(1	$+2+(3+4))+5$	reduce $E \rightarrow \text{num}$
$(E+2+(3+4))+5 \Leftarrow$	(E	$+2+(3+4))+5$	reduce $S \rightarrow E$
$(S+2+(3+4))+5 \Leftarrow$	(S	$+2+(3+4))+5$	shift
$(S+2+(3+4))+5 \Leftarrow$	(S+	$2+(3+4))+5$	shift
$(S+2+(3+4))+5 \Leftarrow$	(S+2	$+(3+4))+5$	reduce $E \rightarrow \text{num}$
$(S+E+(3+4))+5 \Leftarrow$	(S+E	$+(3+4))+5$	reduce $S \rightarrow S+E$
$(S+(3+4))+5 \Leftarrow$	(S	$+(3+4))+5$	shift
$(S+(3+4))+5 \Leftarrow$	(S+	$(3+4))+5$	shift
$(S+(3+4))+5 \Leftarrow$	(S+($3+4))+5$	shift
$(S+(3+4))+5 \Leftarrow$	(S+($+4))+5$	reduce $E \rightarrow \text{num}$

Problem

- How do we know which action to take: whether to shift or reduce, and which production?
- Issues:
 - Sometimes can reduce but shouldn't
 - Sometimes can reduce in different ways

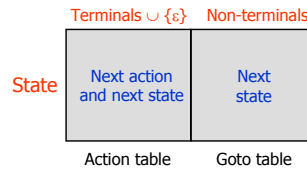
Action Selection Problem

- Given stack σ and look-ahead symbol b , should parser:
 - shift b onto the stack (making it σb)
 - reduce $A \rightarrow \beta$ assuming that stack has the form $\alpha \beta$ (making it αA)

LR Parsing Engine

- **Basic mechanism:**
 - Use a set of **parser states**
 - Use a **stack of states**
 - Use a **parsing table** to:
 - Determine what action to apply (shift/reduce)
 - Determine the next state
- The parser actions can be precisely determined from the table

The LR Parsing Table



- **Algorithm:** look at entry for current state S and input terminal c
 If $\text{Table}[S,c] = s(S')$ then **shift**:
 $\text{push}(S')$
 If $\text{Table}[S,c] = A \rightarrow \alpha$ then **reduce**:
 $\text{pop}(|\alpha|)$; $S' = \text{top}()$; $\text{push}(A)$; $\text{push}(\text{Table}[S',A])$

LR(1) Parsing Table Example

	()	id	,	ϵ	S	L
1	s3		s2			g4	
2	S→id	S→id	S→id	S→id	S→id		
3	s3		s2			g7	g5
4					accept		
5		s6		s8			
6	S→(L	S→(L	S→(L	S→(L	S→(L		
7	L→S	L→S	L→S	L→S	L→S		
8	s3		s2			g9	
9	L→L,S	L→L,S	L→L,S	L→L,S	L→L,S		

LR(k) Grammars

- **LR(k)** = Left-to-right scanning, Right-most derivation, k look-ahead characters
- Main cases: LR(0), LR(1), and some variations (SLR and LALR(1))
- Parsers for **LR(0)** Grammars:
 - Determine the actions without any lookahead symbol
 - Will help us understand shift-reduce parsing