# CS412/413

Introduction to Compilers
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Lecture 23: Control Flow Analysis
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## Outline

- Control flow analysis
  - Detect loops in control flow graphs
  - Dominators
- Loop optimizations
  - Code motion
  - Strength reduction for induction variables
  - Induction variable elimination



## Program Loops

- Loop = a computation repeatedly executed until a terminating condition is reached
- High-level loop constructs:
  - While loop: while(E) S
  - Do-while loop: do S while(E)
  - For loop: for(i=l, i<=u, i+=c) S
- Why are loops important:
  - Most of the execution time is spent in loops
  - Typically: 90/10 rule, 10% code is a loop
- Therefore, loops are important targets of optimizations



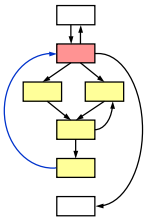
## Detecting Loops

- Need to identify loops in the program
  - Easy to detect loops in high-level constructs
  - Difficult to detect loops in low-level code or in general control-flow graphs
- Examples where loop detection is difficult:
  - Languages with unstructured "goto" constructs: structure of high-level loop constructs may be destroyed
  - Optimizing Java bytecodes (without high-level source program): only low-level code is available



## Control-Flow Analysis

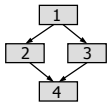
- Goal: identify loops in the control flow graph
- A loop in the CFG:
  - Is a set of CFG nodes (basic blocks)
  - Has a loop header such that control to all nodes in the loop always goes through the header
  - Has a back edge from one of its nodes to the header





## Dominators

- Use concept of dominators to identify loops:
  "CFG node d dominates CFG node n if all the paths from entry node to n go through d"




1 dominates 2, 3, 4
2 doesn't dominate 4
3 doesn't dominate 4

- Intuition:
  - Header of a loop dominates all nodes in loop body
  - Back edges = edges whose heads dominate their tails
  - Loop identification = back edge identification

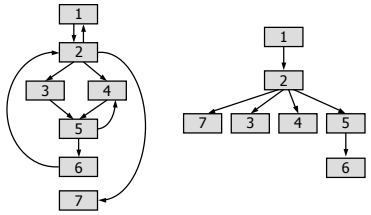
## Immediate Dominators

- Properties:
  1. CFG entry node $n_0$ in dominates all CFG nodes
  2. If d1 and d2 dominate n, then either
  - d1 dominates d2, or
  - d2 dominates d1
- Immediate dominator idom(n) of node n:
  - idom(n) ≠ n
  - idom(n) dominates n
  - If m dominates n, then m dominates idom(n)
- Immediate dominator idom(n) exists and is unique because of properties 1 and 2



## Dominator Tree

- Build a dominator tree as follows:
  - Root is CFG entry node $n_0$
  - m is child of node n iff n=idom(m)
- Example:






## Computing Dominators

- Formulate problem as a system of constraints:
  - dom(n) is set of nodes who dominate n
  - dom($n_0$)= {$n_0$}
  - dom(n) = ∩ { dom(m) | m ∈ pred(n) }
- Can also formulate problem in the dataflow framework
  - What is the dataflow information?
  - What is the lattice?
  - What are the transfer functions?
  - Use dataflow analysis to compute dominators



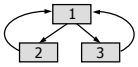
## Natural Loops

- Back edge: edge n→h such that h dominates n
- Natural loop of a back edge n→h:
  - h is loop header
  - Loop nodes is set of all nodes that can reach n without going through h
- Algorithm to identify natural loops in CFG:
  - Compute dominator relation
  - Identify back edges
  - Compute the loop for each back edge



## Disjoint and Nested Loops

- Property: for any two natural loops in the flow graph, one of the following is true:
  1. They are disjoint
  2. They are nested
  3. They have the same header
- Eliminate alternative 3: if two loops have the same header and none is nested in the other, combine all nodes into a single loop

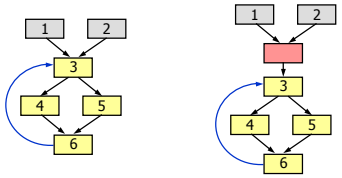



Two loops: {1,2} and {1,3}
Combine into one loop: {1,2,3}



## Loop Preheader

- Several optimizations add code before header
- Insert a new basic block (called preheader) in the CFG to hold this code



## Loop optimizations

- Now we know the loops in the program
- Next: optimize loops
  - Loop invariant code motion
  - Strength reduction of induction variables
  - Induction variable elimination



## Loop Invariant Code Motion

- Idea: if a computation produces same result in all loop iterations, move it out of the loop
- Example:
```
for (i=0; i<10; i++)
    a[i] = 10*i + x*x;
```
- Expression x*x produces the same result in each iteration; move it of the loop:
```
t = x*x;
for (i=0; i<10; i++)
    a[i] = 10*i + t;
```



## Loop Invariant Computation

- An instruction a = b OP c is loop-invariant if each operand is:
  - Constant, or
  - Has all definitions outside the loop, or
  - Has exactly one definition, and that is a loop-invariant computation
- Reaching definitions analysis computes all the definitions of x and y which may reach t = x OP y



## Algorithm

INV = ∅

Repeat

  for each instruction i ∉ INV

    if operands are constants, or

      have definitions outside the loop, or

      have exactly one definition d ∈ INV

    then INV = INV U {i}

Until no changes in INV



## Code Motion

- Next: move loop-invariant code out of the loop
- Suppose a = b OP c is loop-invariant
- We want to hoist it out of the loop
- Code motion of a definition d: a = b OP c in pre-header is valid if:
  1. Definition d dominates all loop exits where a is live
  2. There is no other definition of a in loop
  3. All uses of a in loop can only be reached from definition d



## Other Issues

- Preserve dependencies between loop-invariant instructions when hoisting code out of the loop

```
for (i=0; i<N; i++) {
  x = y+z;
  a[i] = 10*i + x*x;
}
```

```
x = y+z;
t = x*x;
for(i=0; i<N; i++)
  a[i] = 10*i + t;
```

- Nested loops: apply loop invariant code motion algorithm multiple times

```
for (i=0; i<N; i++)
  for (j=0; j<M; j++)
    a[i][j] = x*x + 10*i + 100*j;
```

```
t1 = x*x;
for (i=0; i<N; i++) {
  t2 = t1+ 10*i;
  for (j=0; j<M; j++)
    a[i][j] = t2 + 100*j; }
```