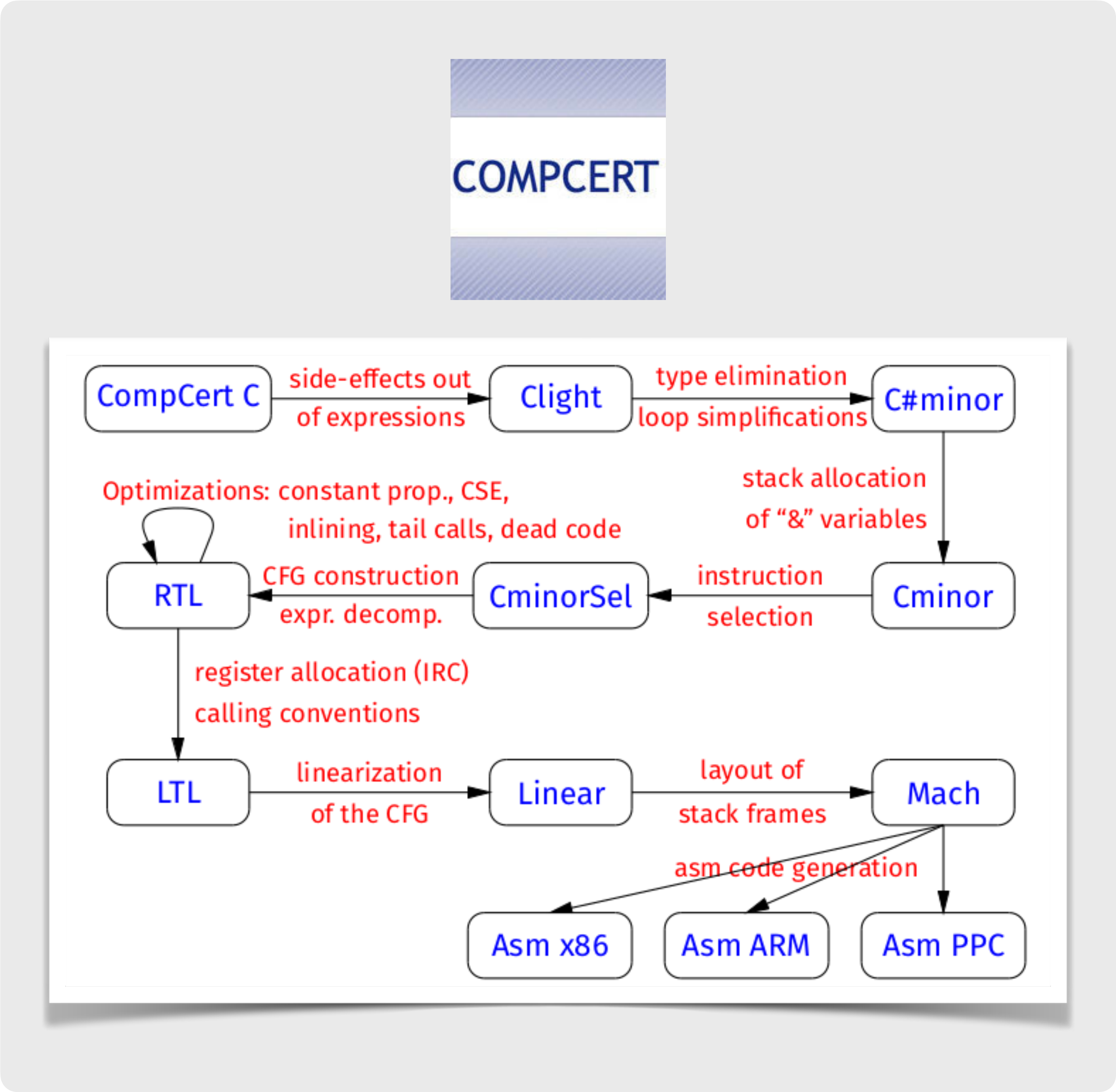
CS 4110
Victory Lap

Nate Foster
Cornell

A “golden age” of formal methods



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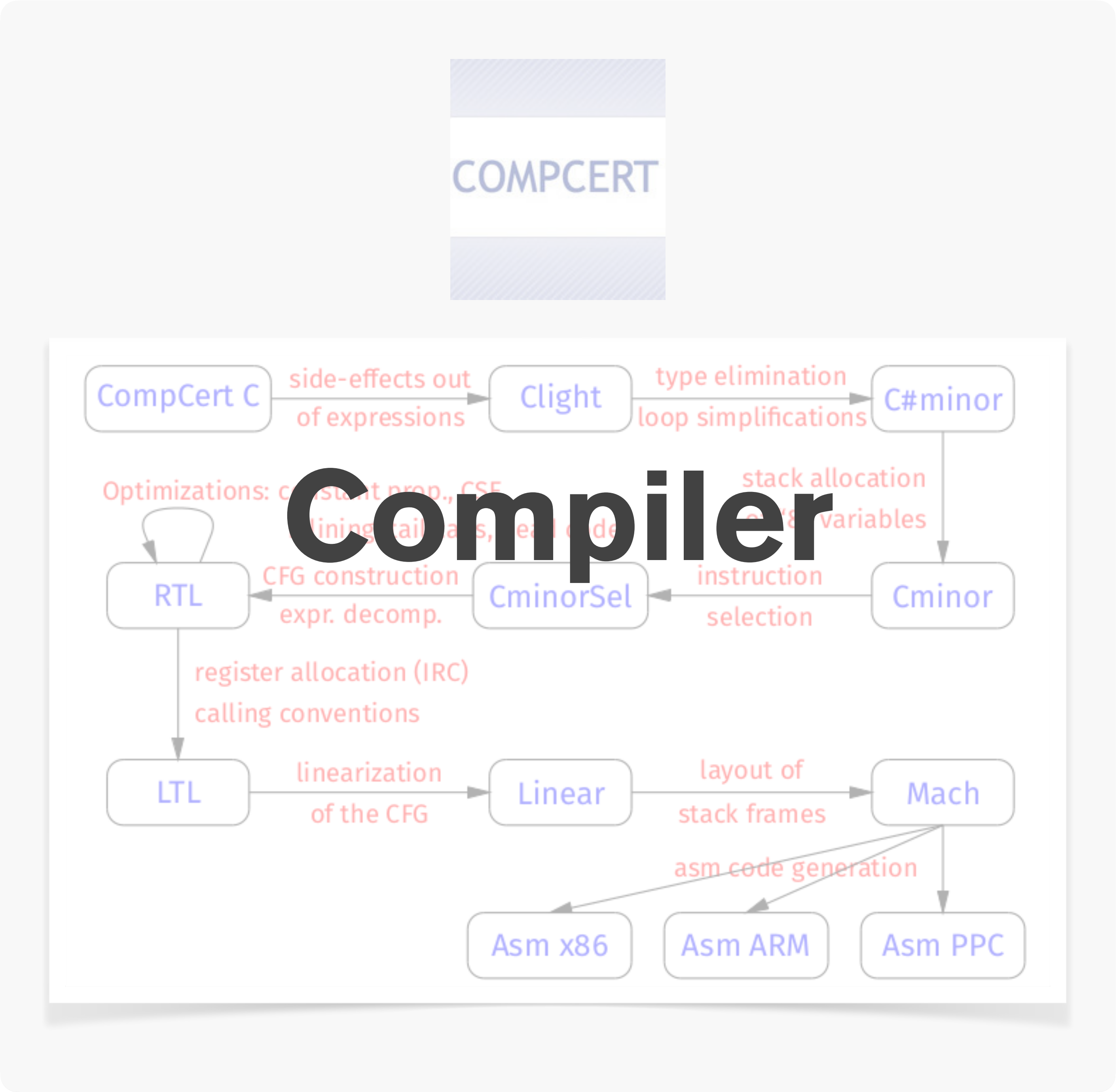

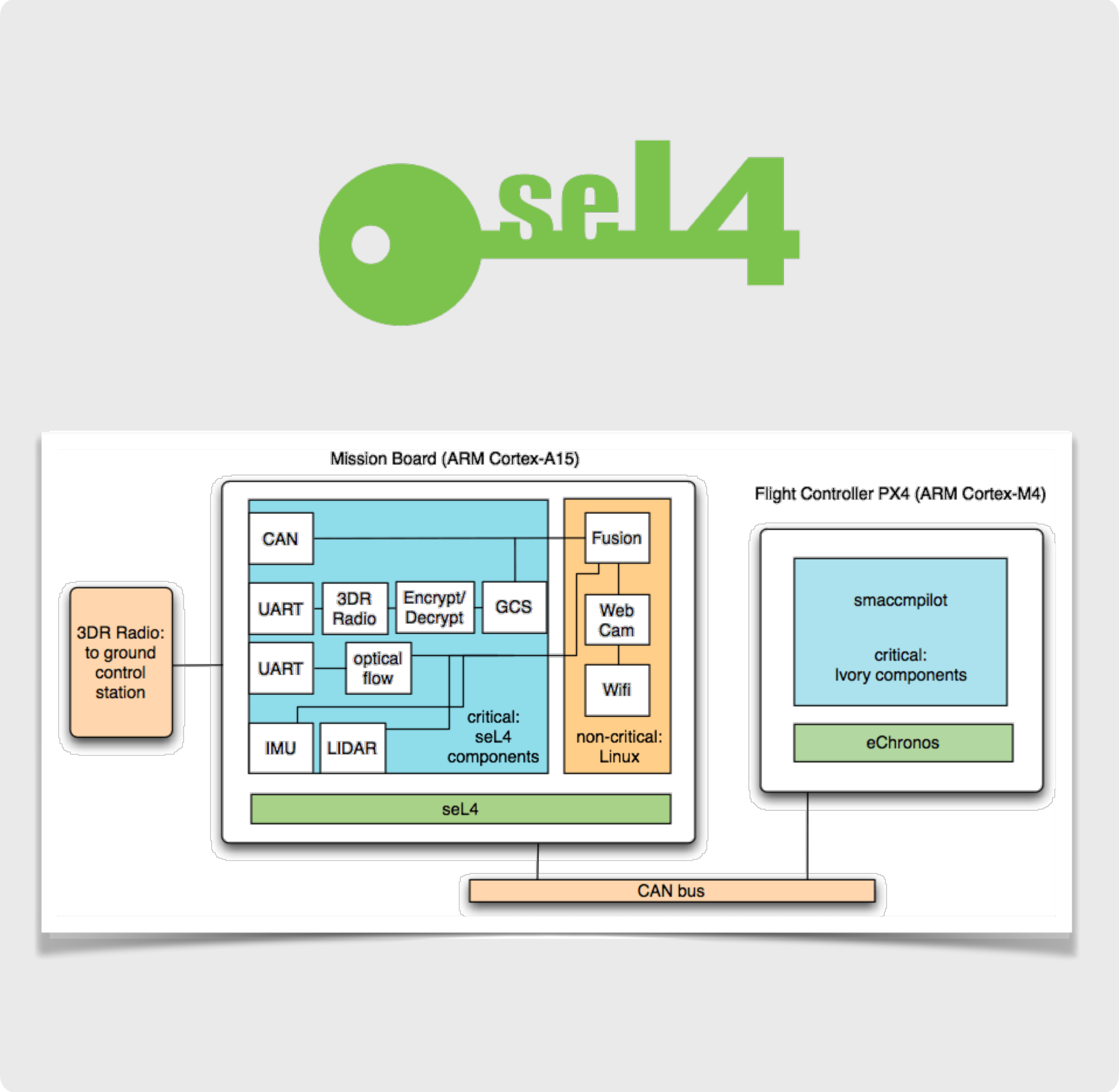

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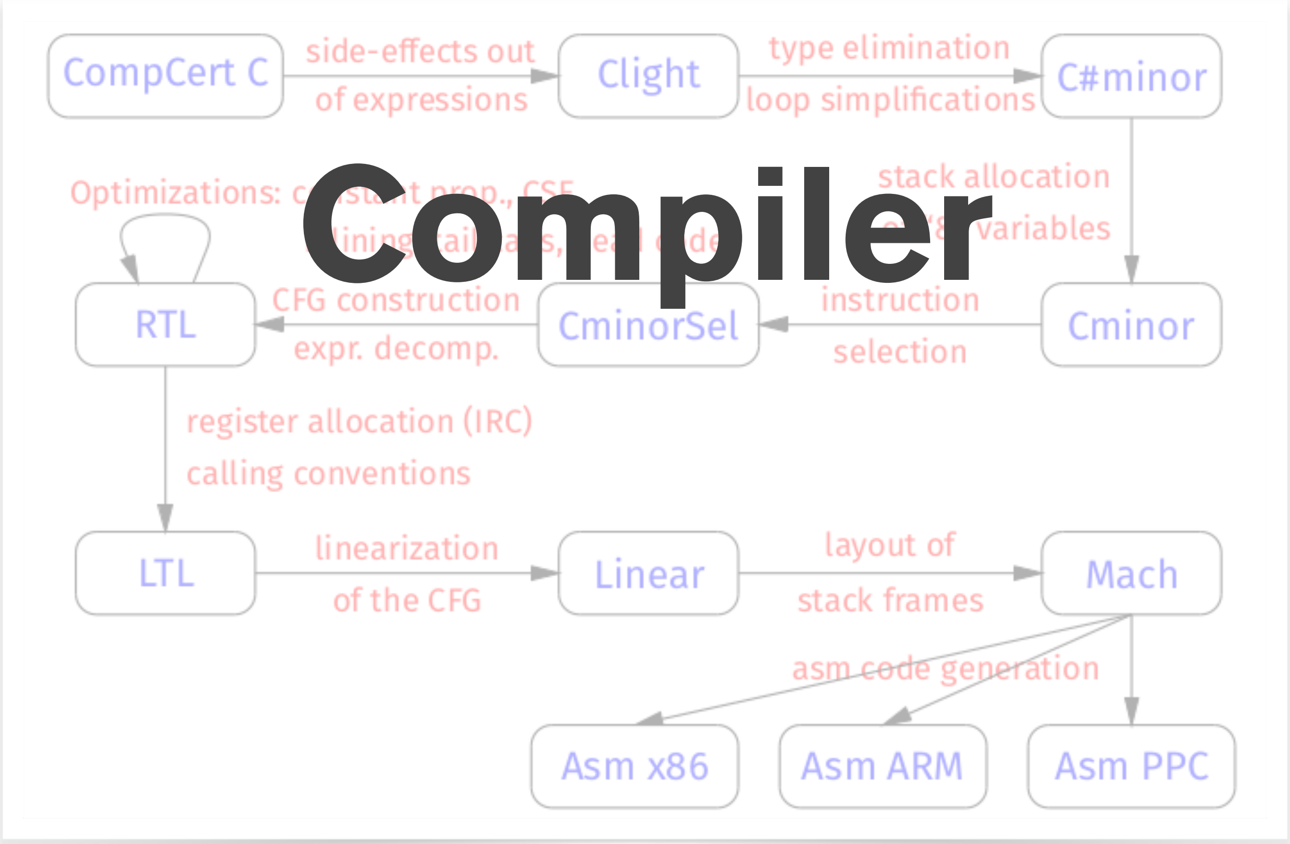
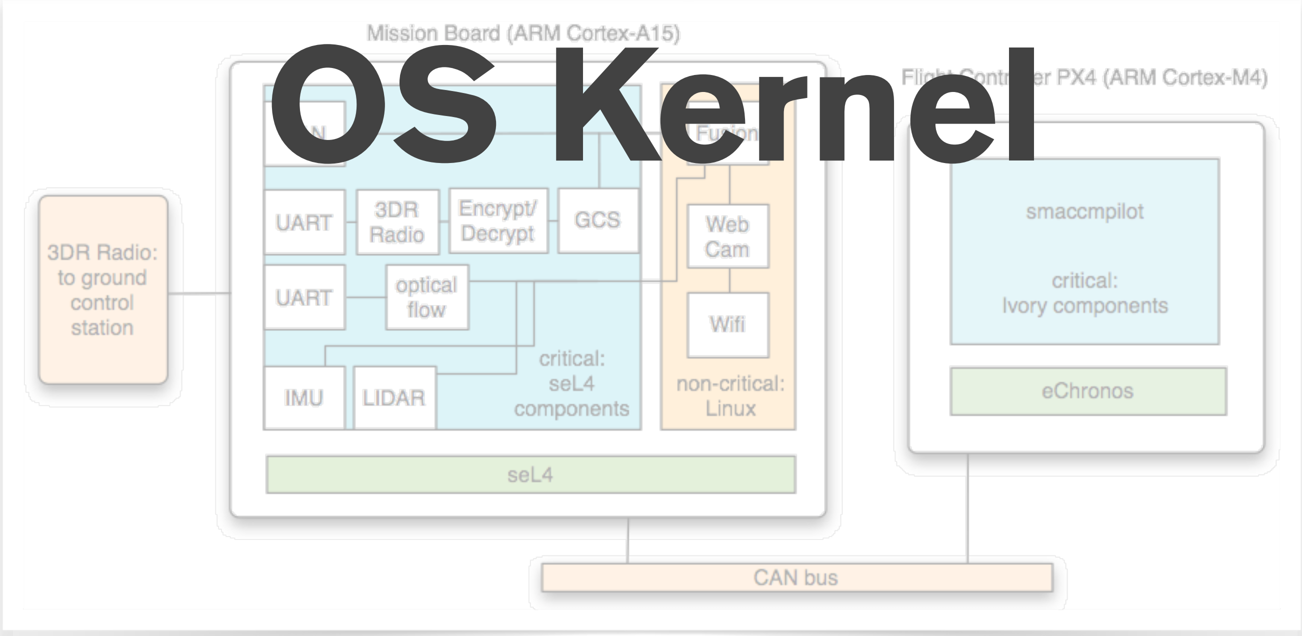
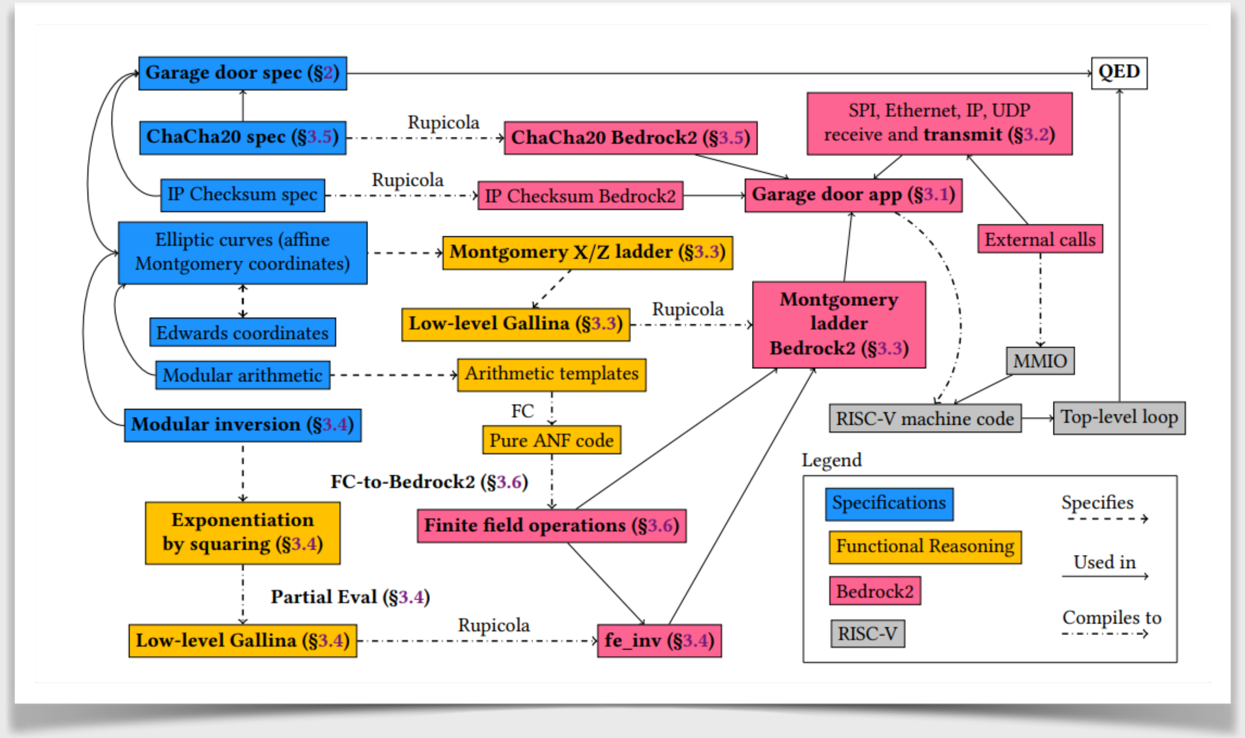


Compiler



OS Kernel



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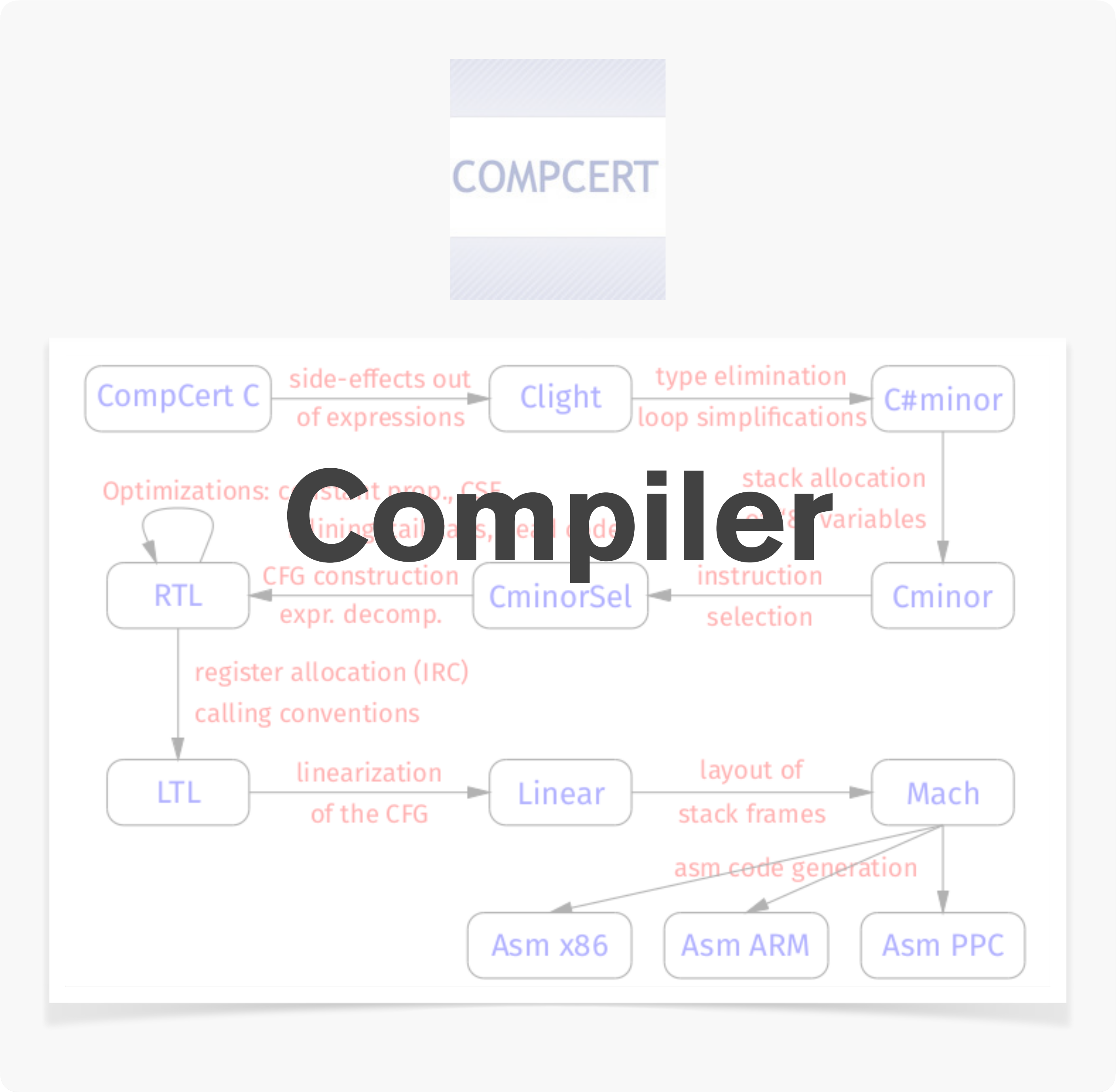

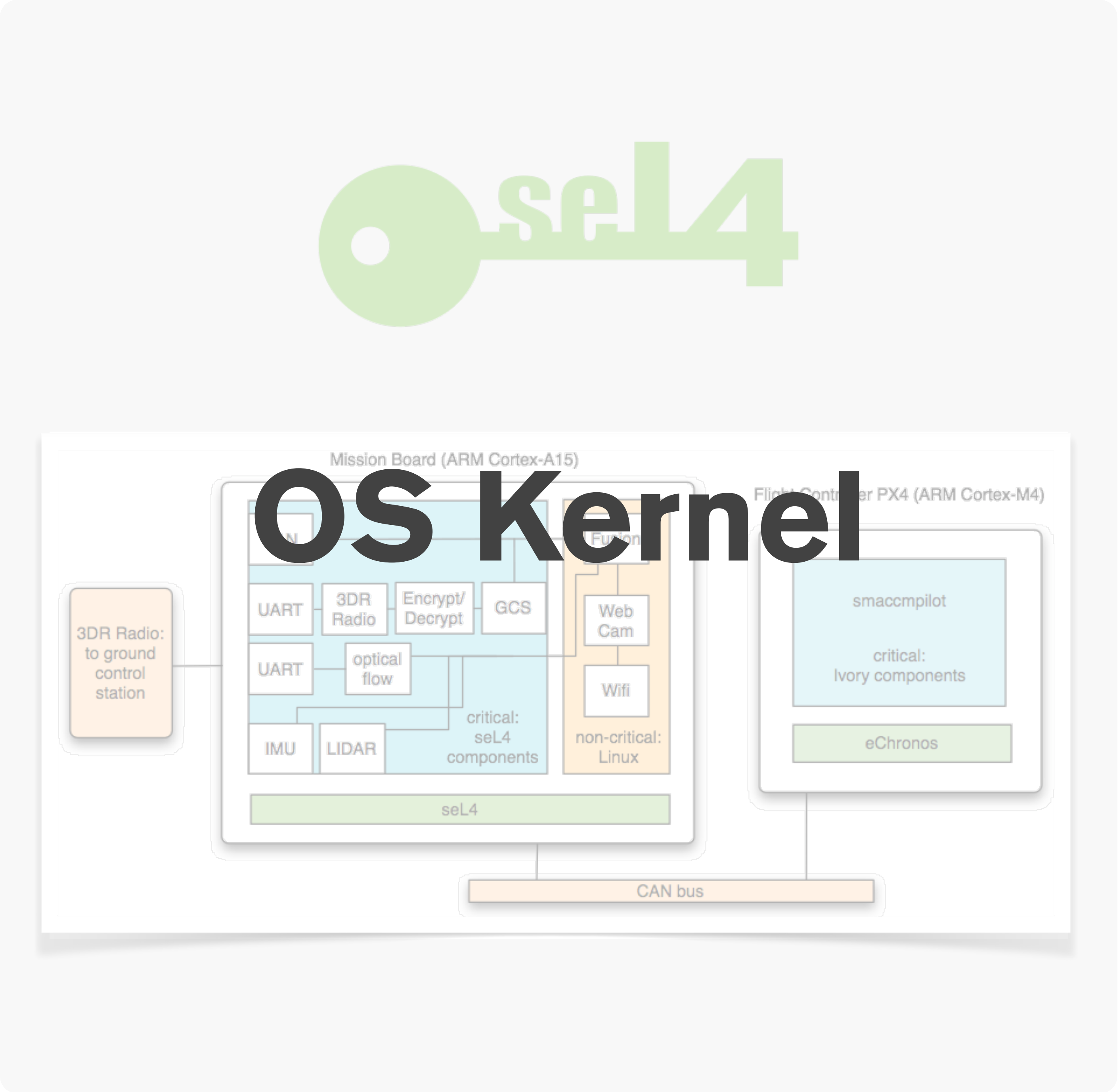

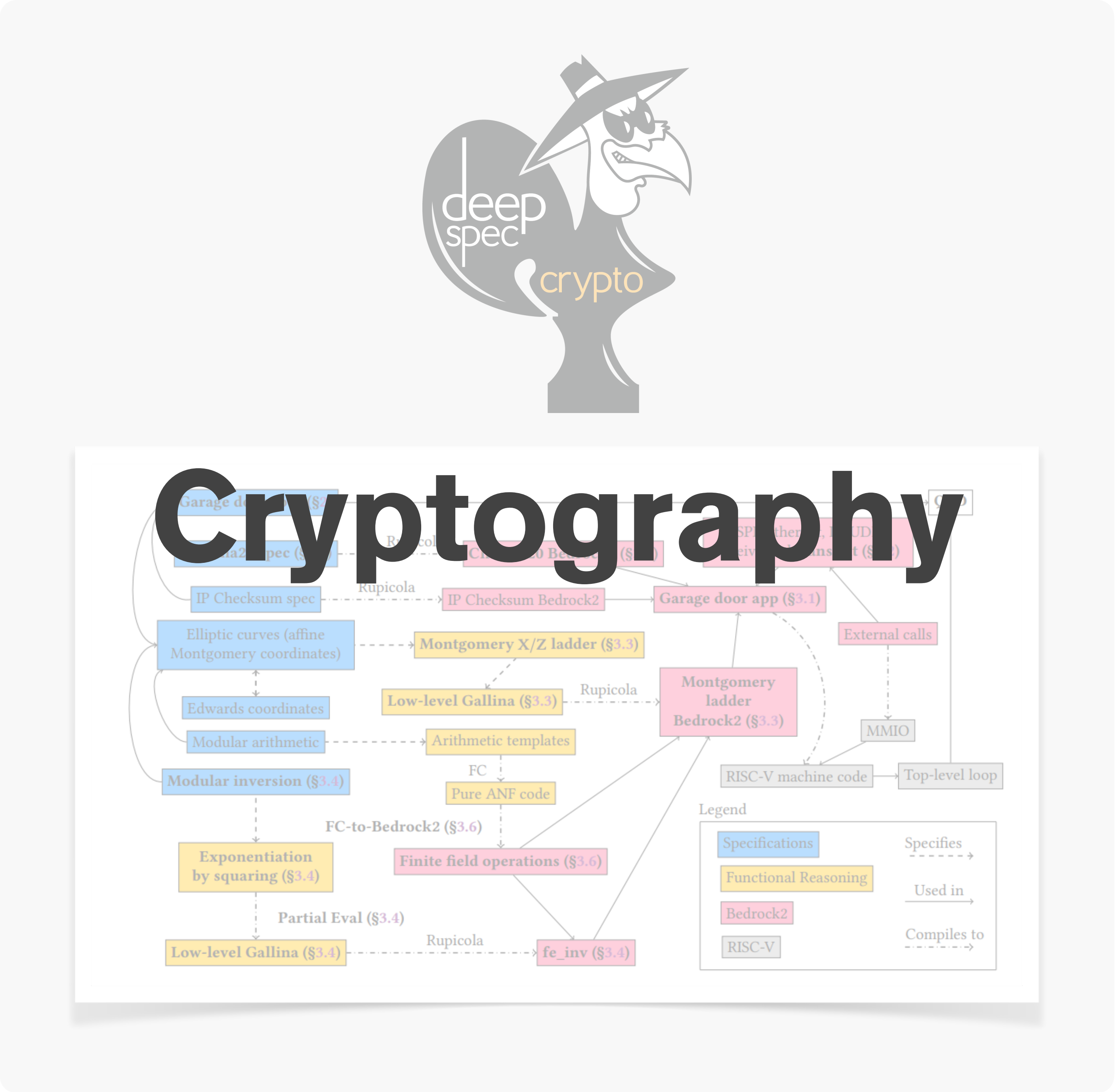

What about networks?

THE DESIGN PHILOSOPHY OF THE DARPA INTERNET PROTOCOLS

David D. Clark

Massachusetts Institute of Technology
Laboratory for Computer Science
Cambridge, Ma. 02139

Abstract

The Internet protocol suite, TCP/IP, was first proposed fifteen years ago. It was developed by the Defense Advanced Research Projects Agency (DARPA), and has been used widely in military and commercial systems. While there have been papers and specifications that describe how the protocols work, it is sometimes difficult to deduce from these why the protocol is as it is. For example, the Internet protocol is based on a connectionless or datagram mode of service. The motivation for this has been greatly misunderstood. This paper attempts to capture some of the early reasoning which shaped the Internet protocols.

1. Introduction

For the last 15 years[1], the Advanced Research Projects Agency of the U.S. Department of Defense has been developing a suite of protocols for packet switched networking. These protocols, which include the Internet Protocol (IP), and the Transmission Control Protocol (TCP), are now U.S. Department of Defense standards for internetworking, and are in wide use in the commercial networking environment. The ideas developed in this effort have also influenced other protocol suites, most importantly the connectionless configuration of the ISO protocols[2, 3, 4].

While specific information on the DOD protocols is fairly generally available[5, 6, 7], it is sometimes difficult to determine the motivation and reasoning which led to the design.

In fact, the design philosophy has evolved considerably from the first proposal to the current standards. For example, the idea of the datagram, or connectionless service, does not receive particular emphasis in the first paper, but has come to be the defining characteristic of the protocol. Another example is the layering of the architecture into the IP and TCP layers. This seems basic to the design, but was also not a part of the original proposal. These changes in the Internet design arose through the repeated pattern of implementation and testing that occurred before the standards were set.

The Internet architecture is still evolving. Sometimes a new extension challenges one of the design principles, but in any case an understanding of the history of the design provides a necessary context for current design extensions. The connectionless configuration of ISO protocols has also been colored by the history of the Internet suite, so an understanding of the Internet design philosophy may be helpful to those working with ISO.

This paper catalogs one view of the original objectives of the Internet architecture, and discusses the relation between these goals and the important features of the protocols.

2. Fundamental Goal

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This paper catalogs one view of the original objectives of the Internet architecture, and discusses the relation between these goals and the important features of the protocols.

2. Fundamental Goal

"While **tools to verify logical correctness are useful**, both at the specification and implementation stage, **they do not help with the severe problems that often arise related to performance.**"

Evolution of networks

Conventional Networks

- Vertically integrated
- Fixed protocols
- Vendors write the software

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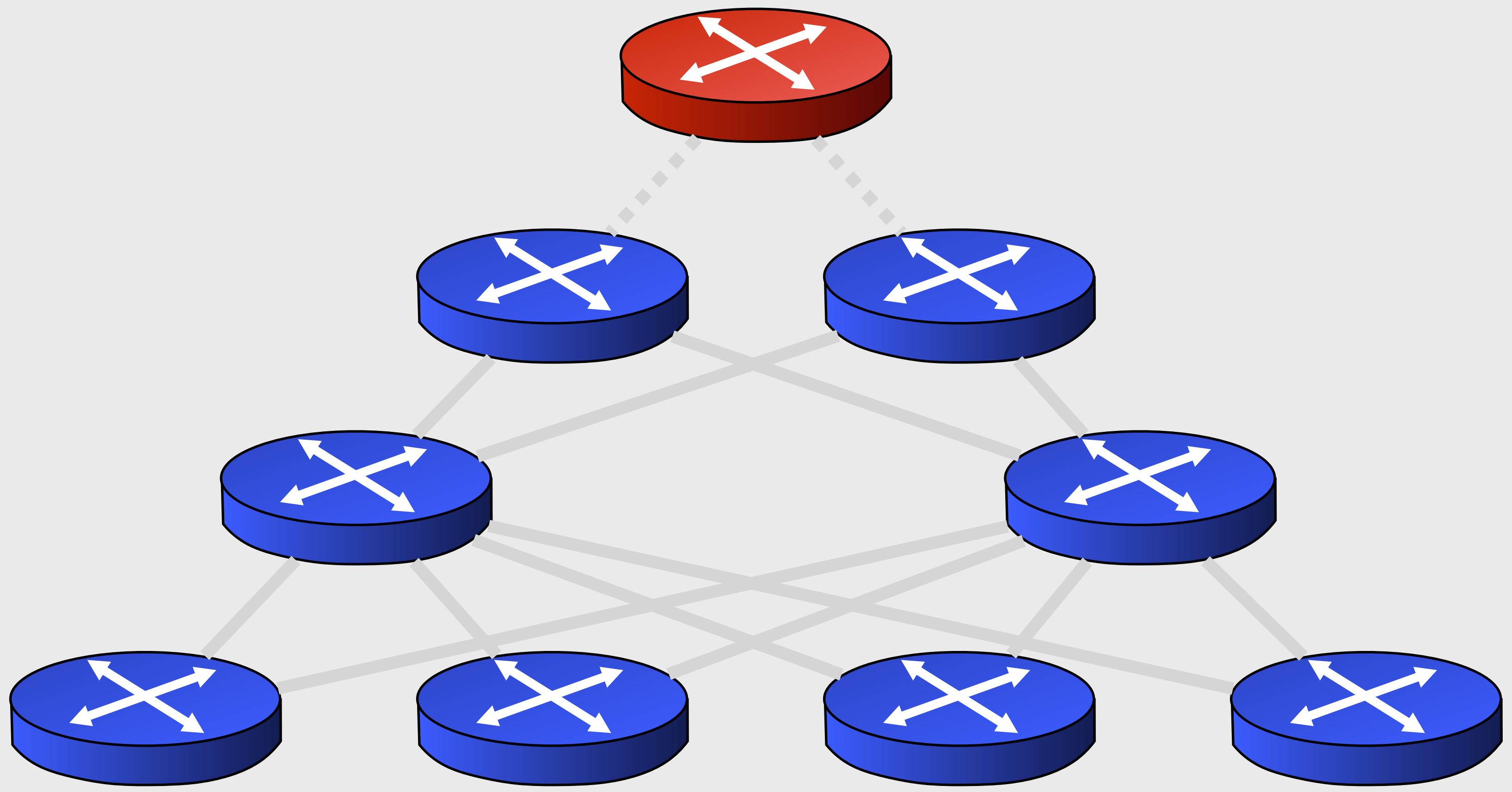
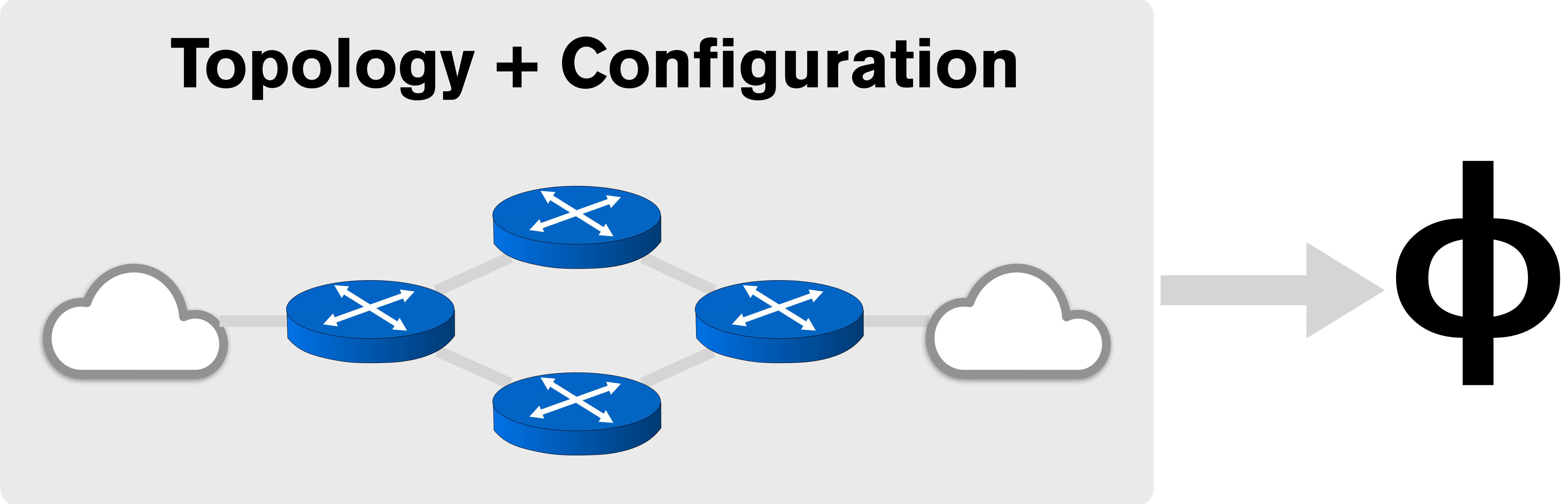
Modern Networks



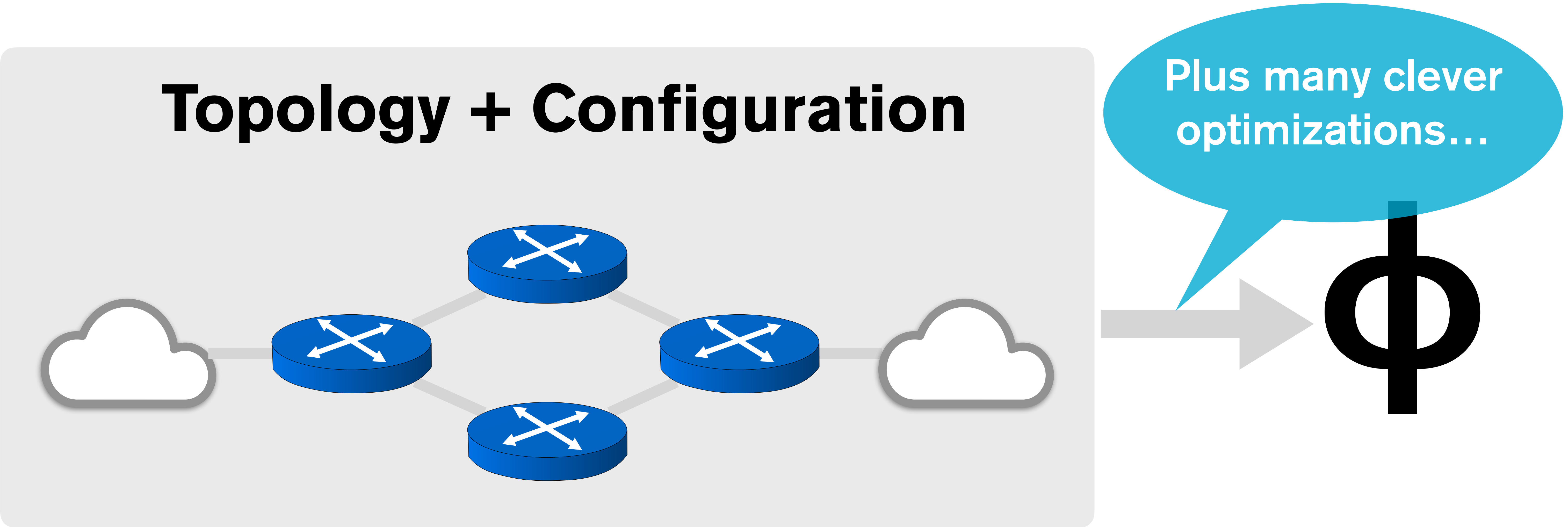
- Disaggregated
- Programmable
- Owners write the software

Status quo: solver-based approach

Status quo: solver-based approach

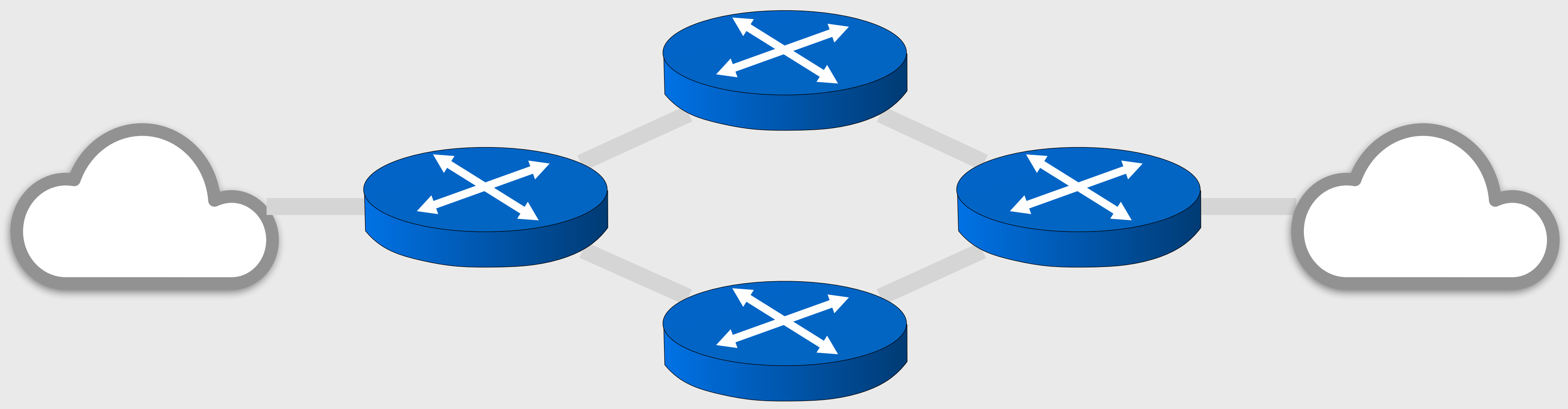


Status quo: solver-based approach



Status quo: solver-based approach

Topology + Configuration



Plus many clever optimizations…

→ ϕ

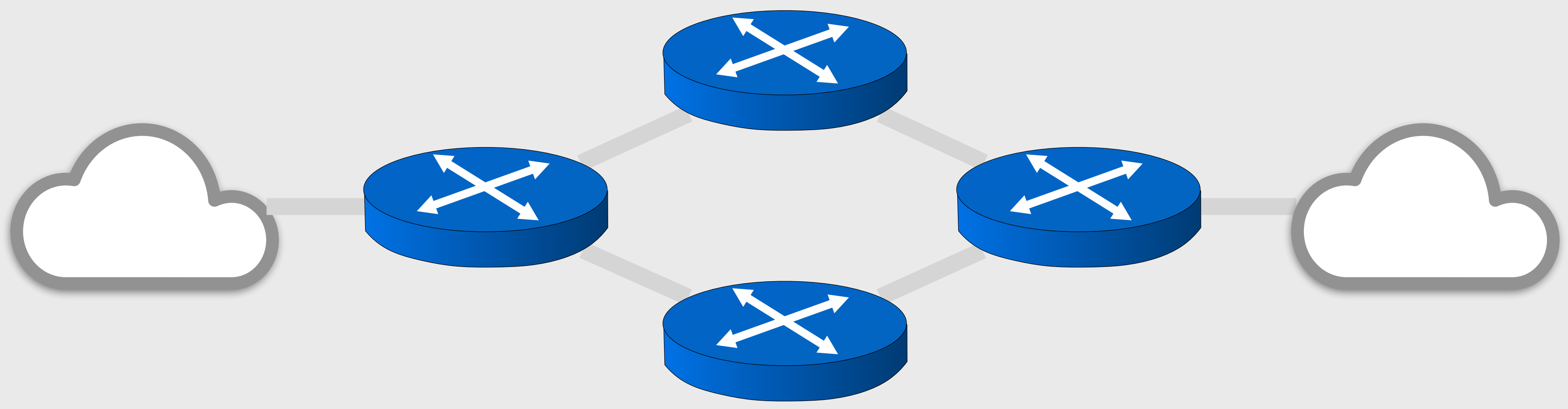
Specification

"The network should be free of forwarding loops, and every packet should be delivered to its destination"

→ ψ

Status quo: solver-based approach

Topology + Configuration



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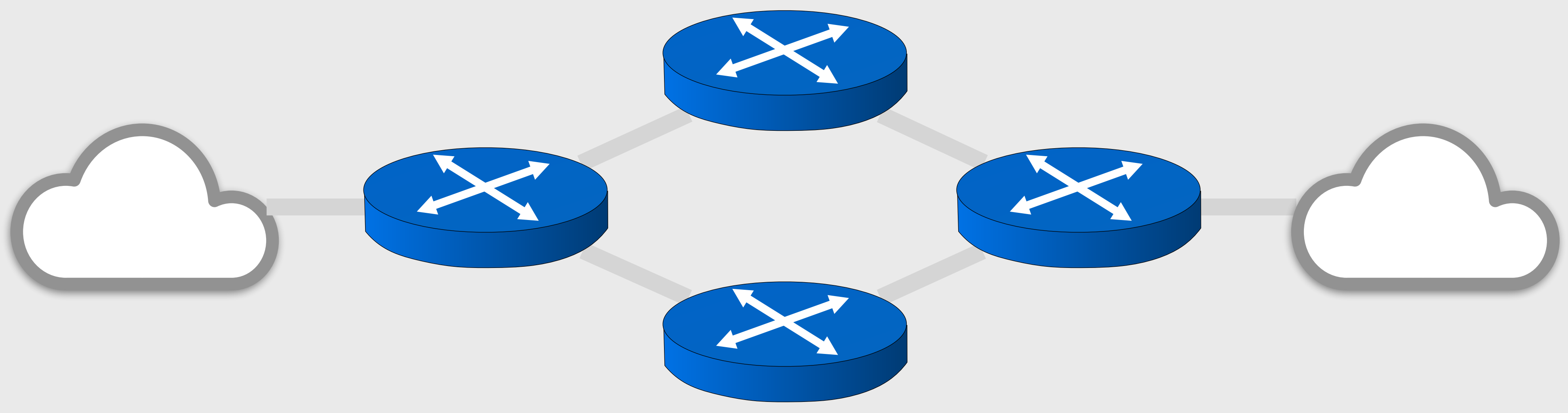
Ψ

SMT Solver

Status quo: solver-based approach

Topology + Configuration



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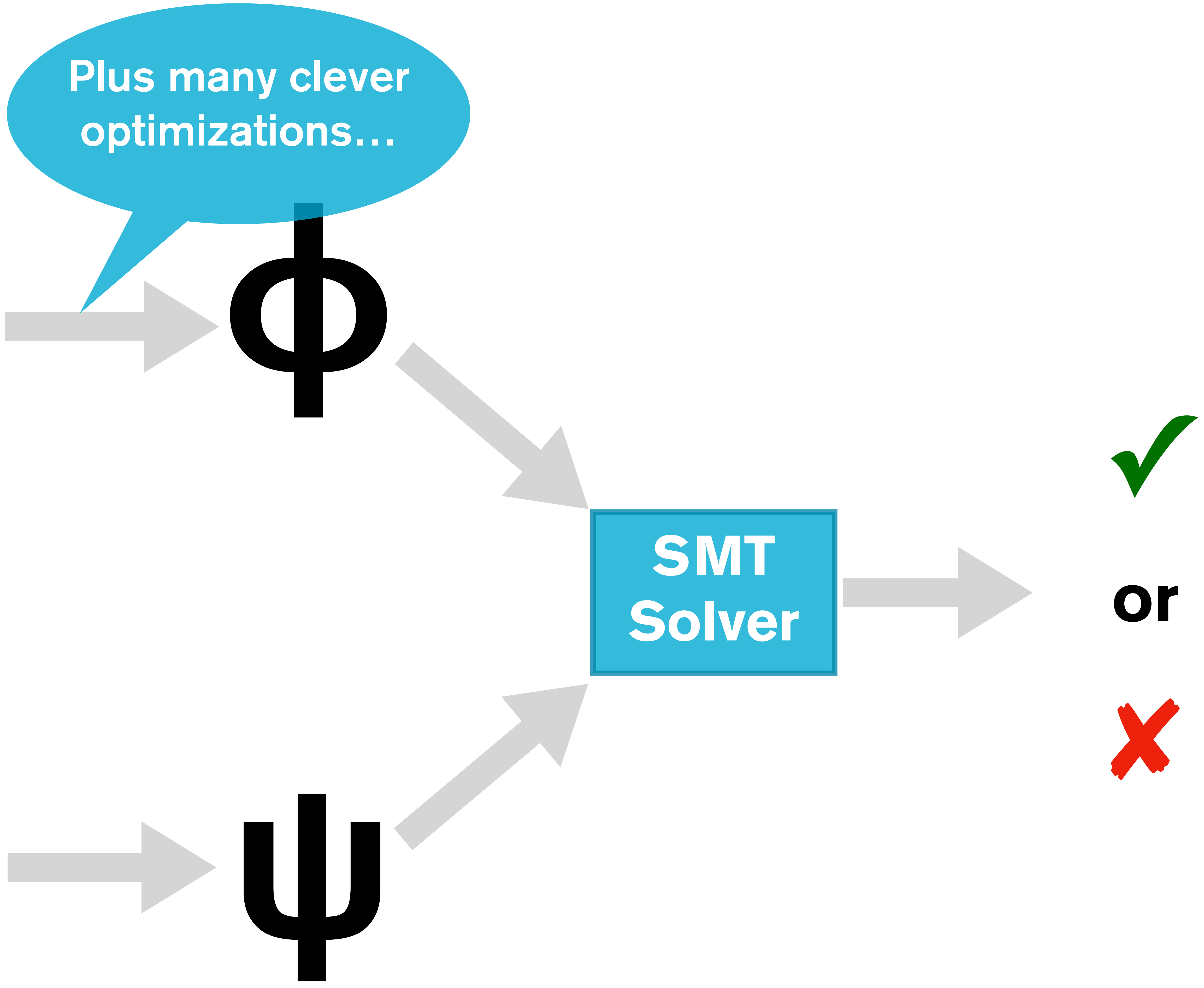
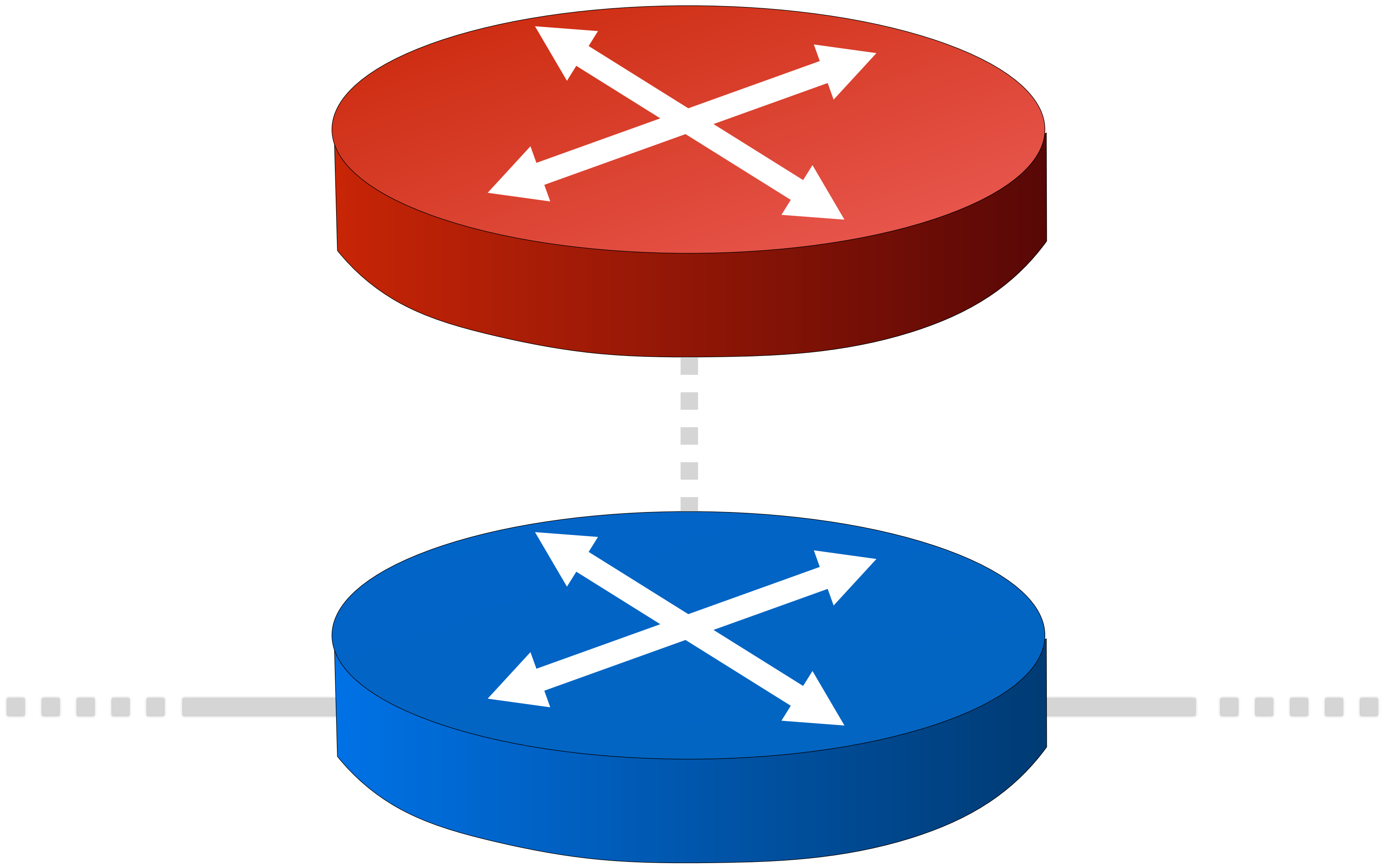
“The network should be free of forwarding loops, and every packet should be delivered to its destination”

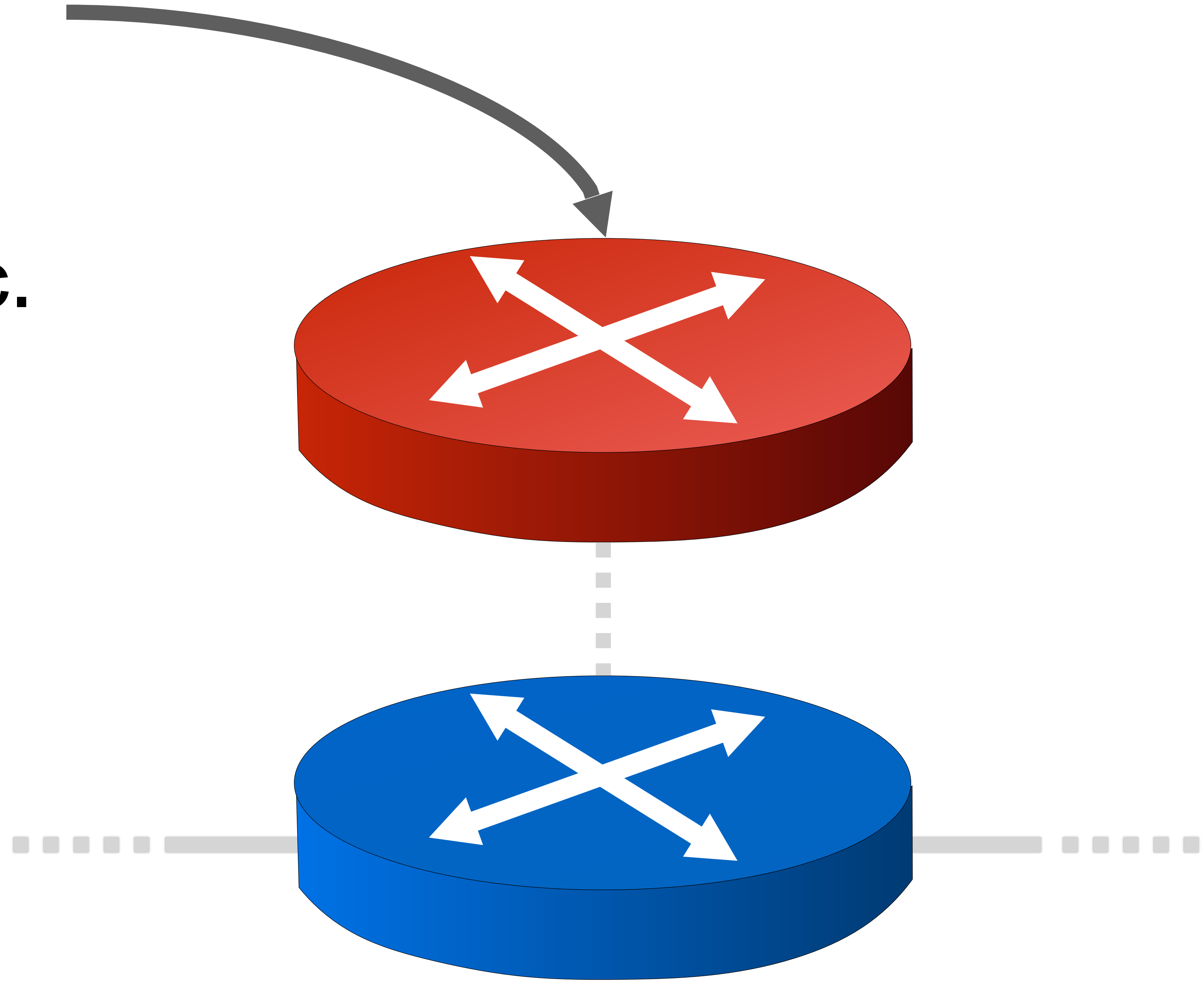


Networking terminology



Networking terminology

Control Plane
discovers topology,
computes routes,
manages policy, etc.



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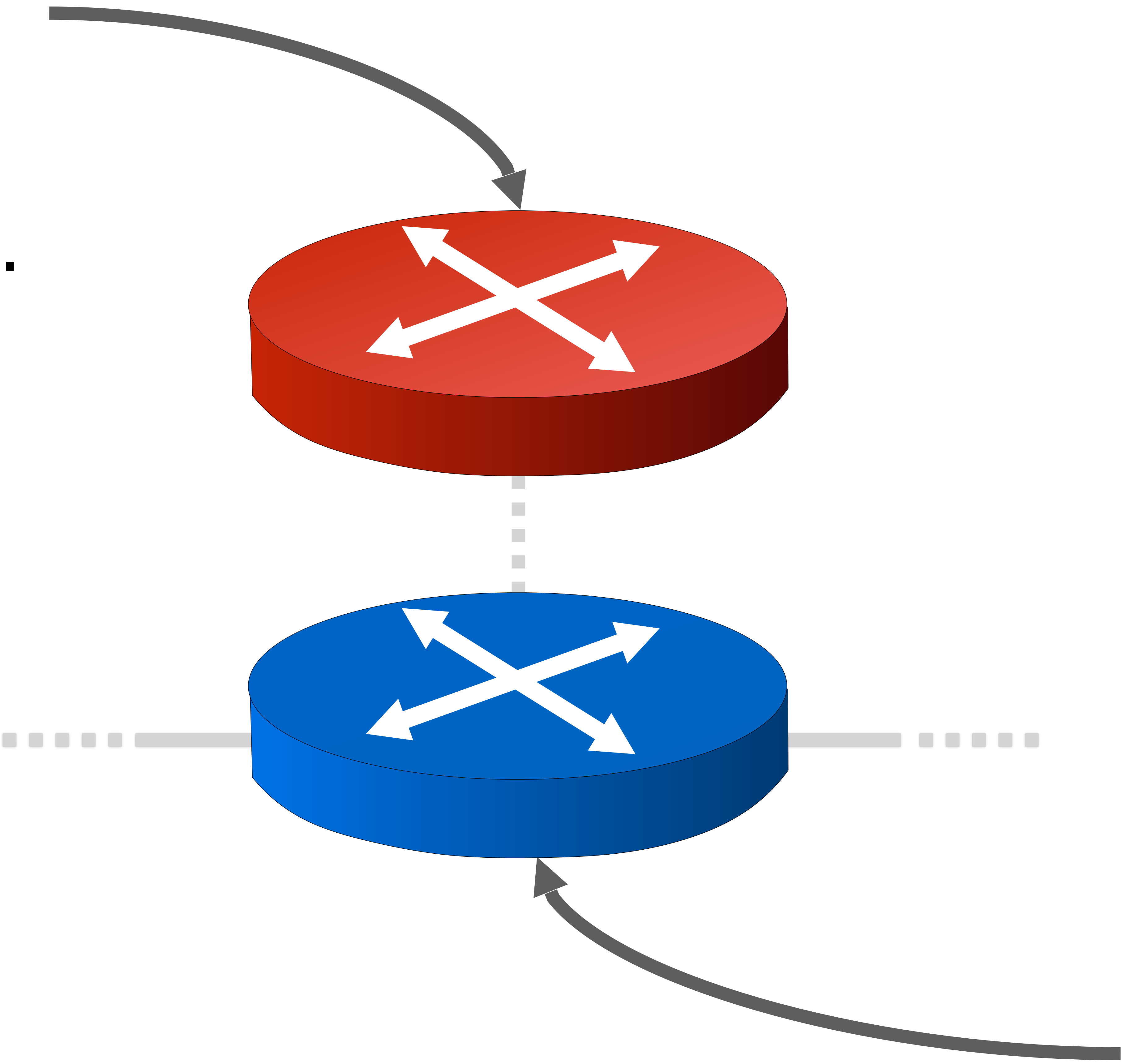
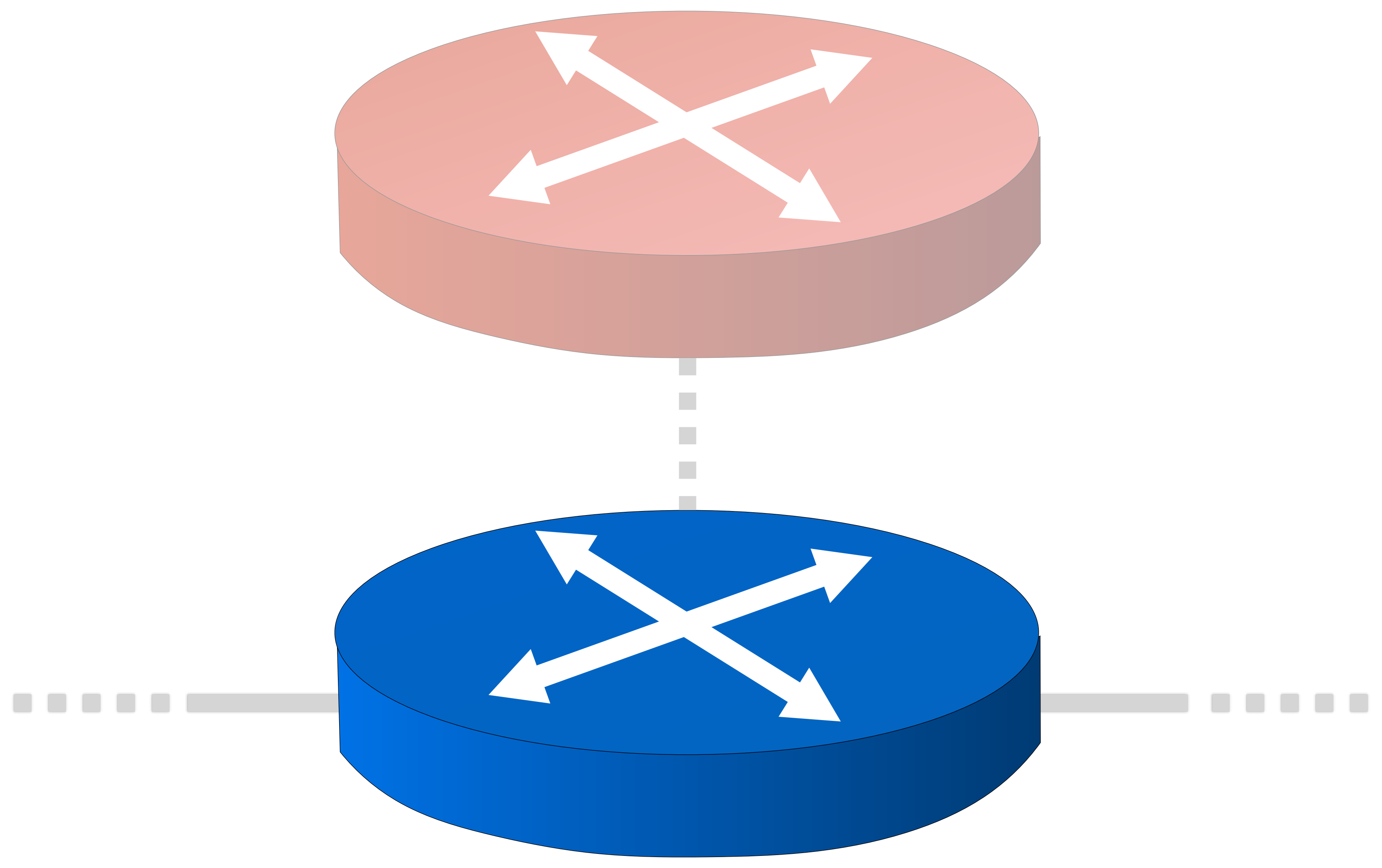


Data plane
forwards packets,
monitors traffic,
enforces access
control, etc.

Data Plane



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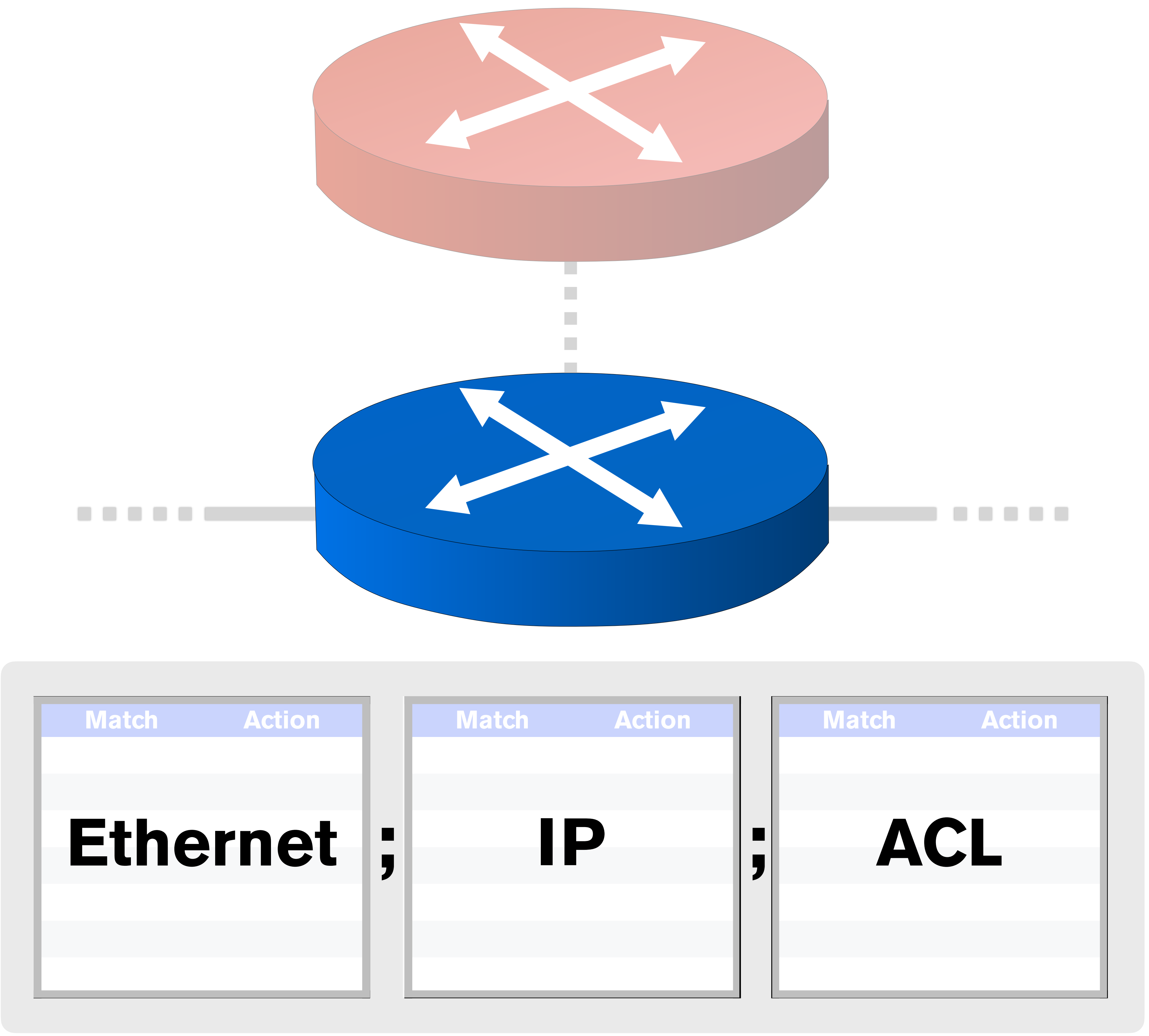

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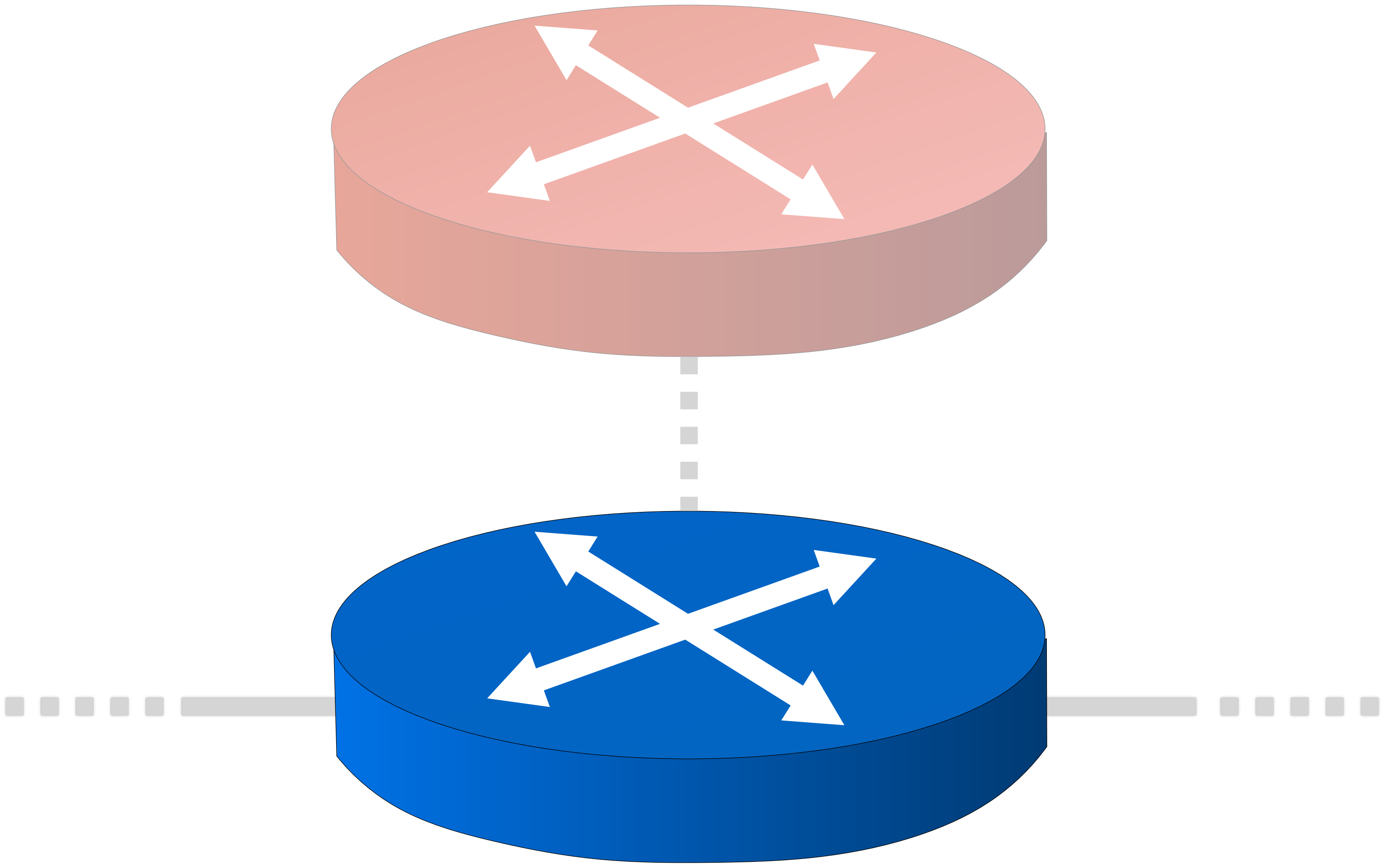
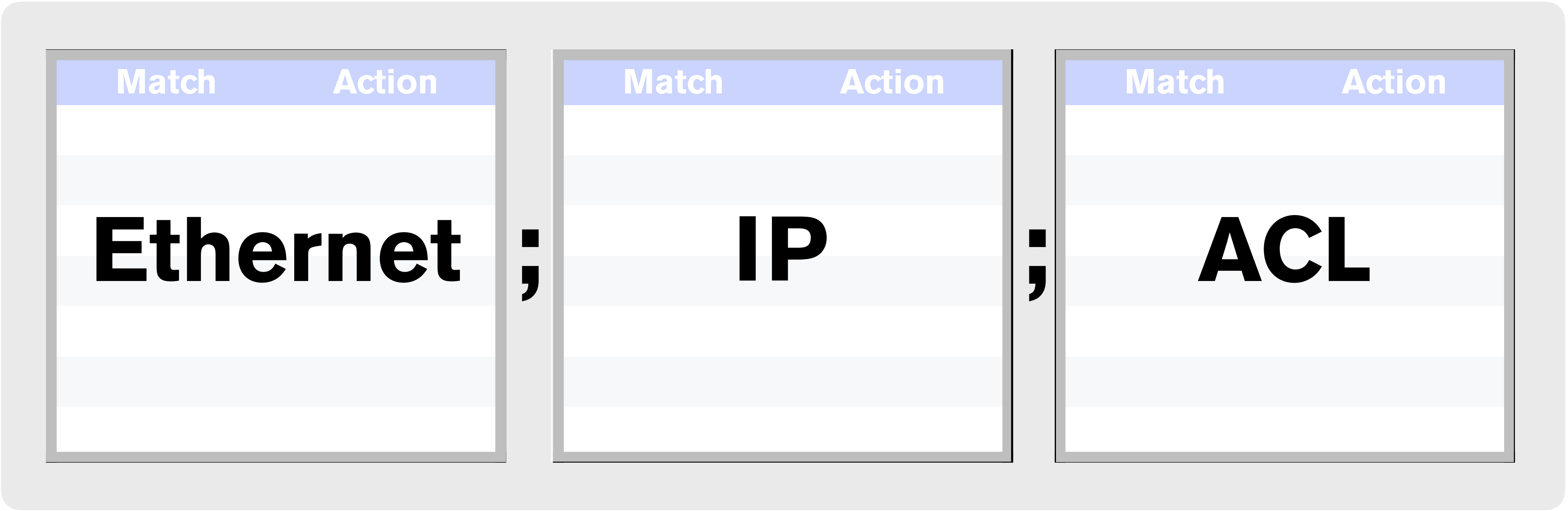
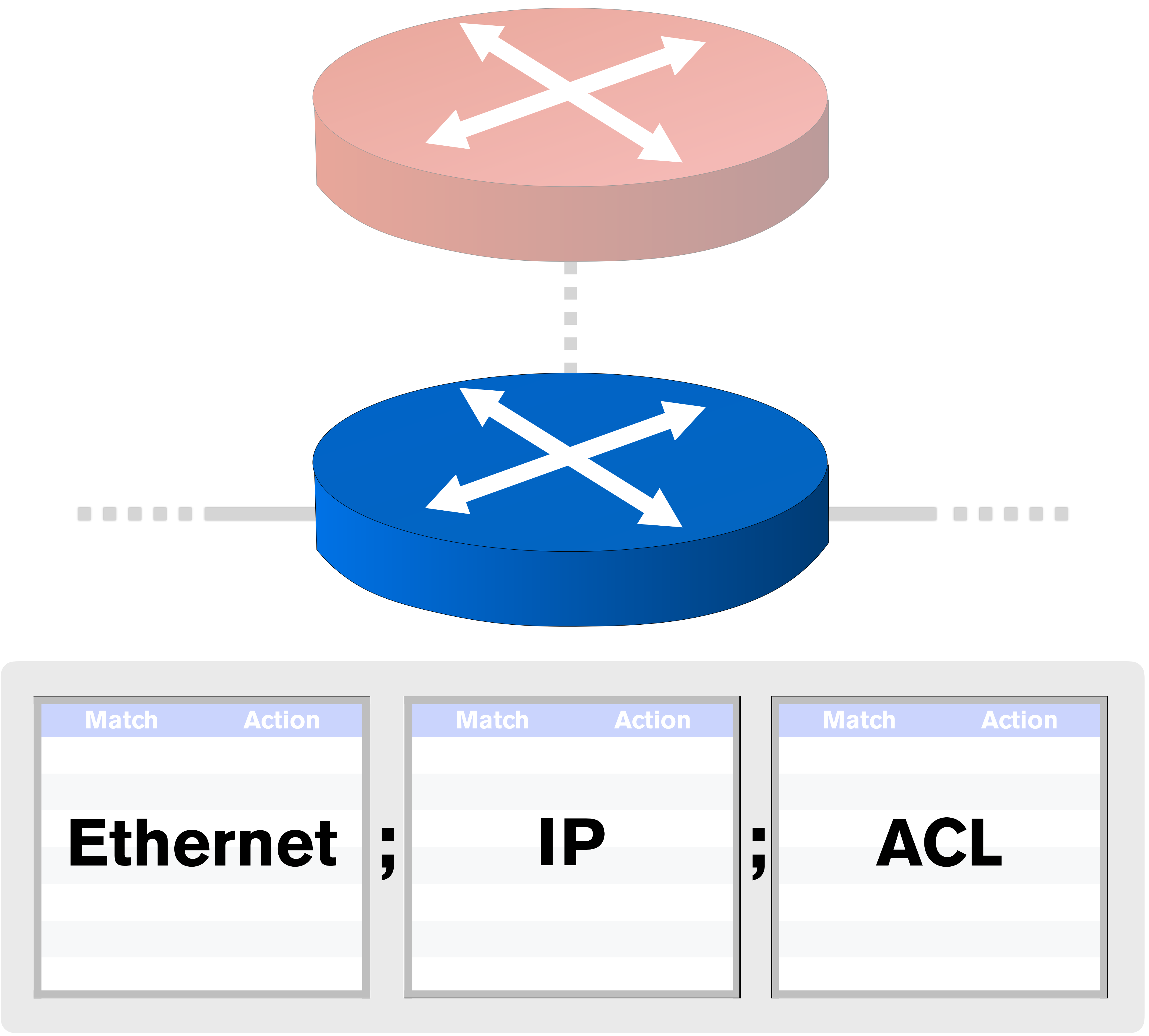




Typically structured as a *pipeline* of *match-action forwarding tables*, in hardware or software

Data Plane



Typically structured as a *pipeline* of *match-action forwarding tables*, in hardware or software

Each table contains *rules* that:

- *Match* on packet headers
- Execute *Actions* that transform, forward, or drop packets

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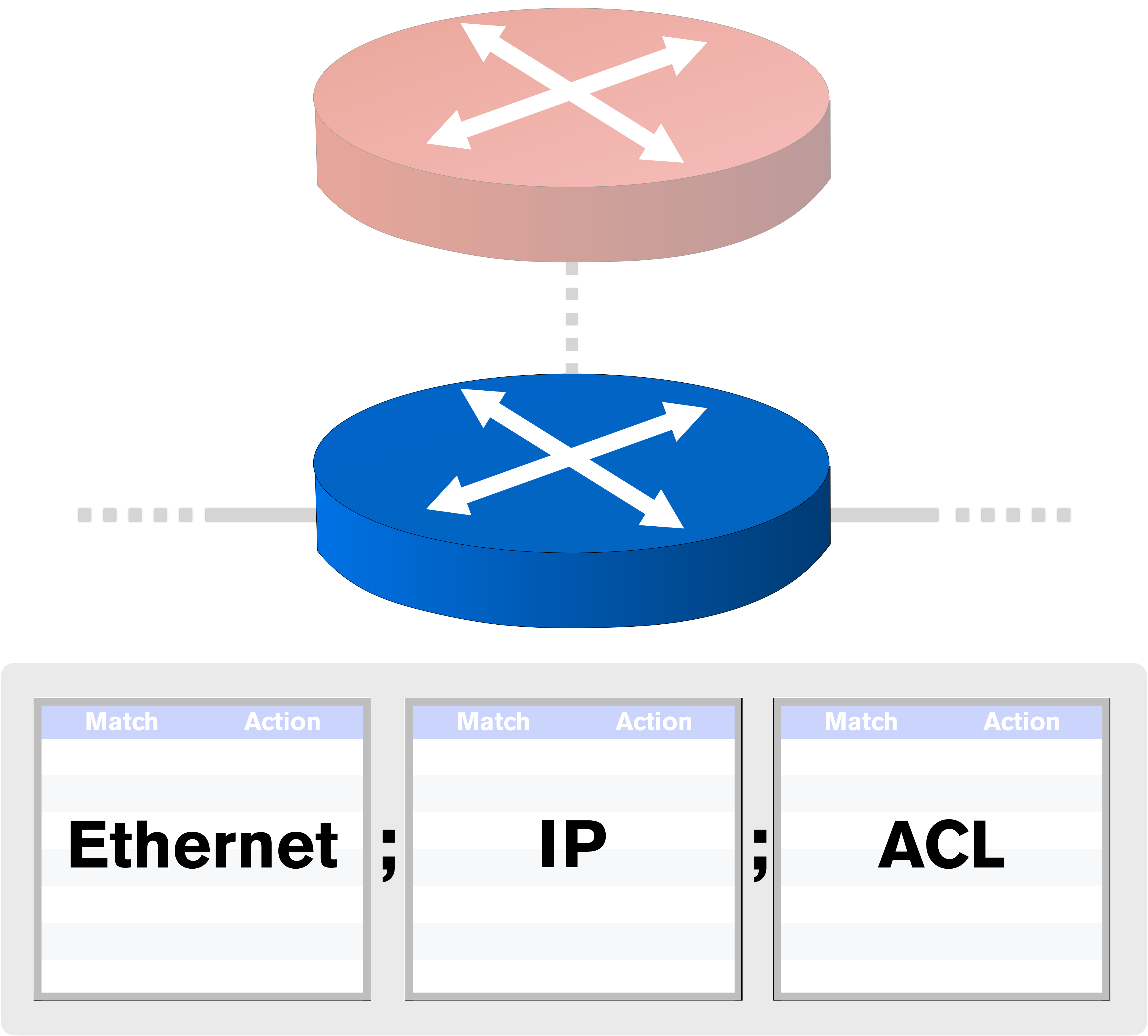

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Each table contains *rules* that:

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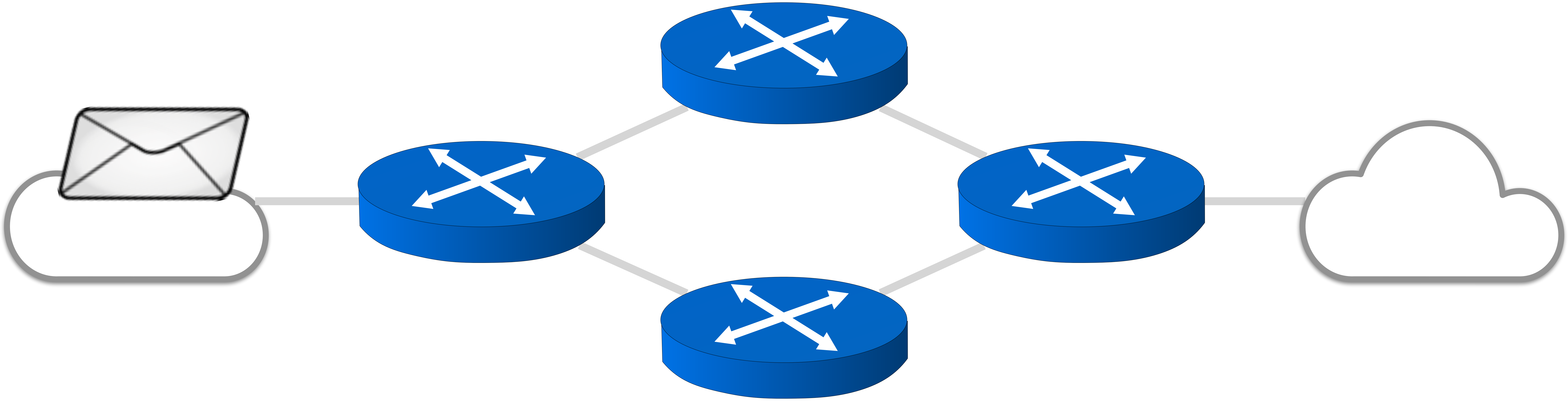
Control plane can *dynamically reconfigure* the network by modifying the rules in tables

Data Plane Behavior



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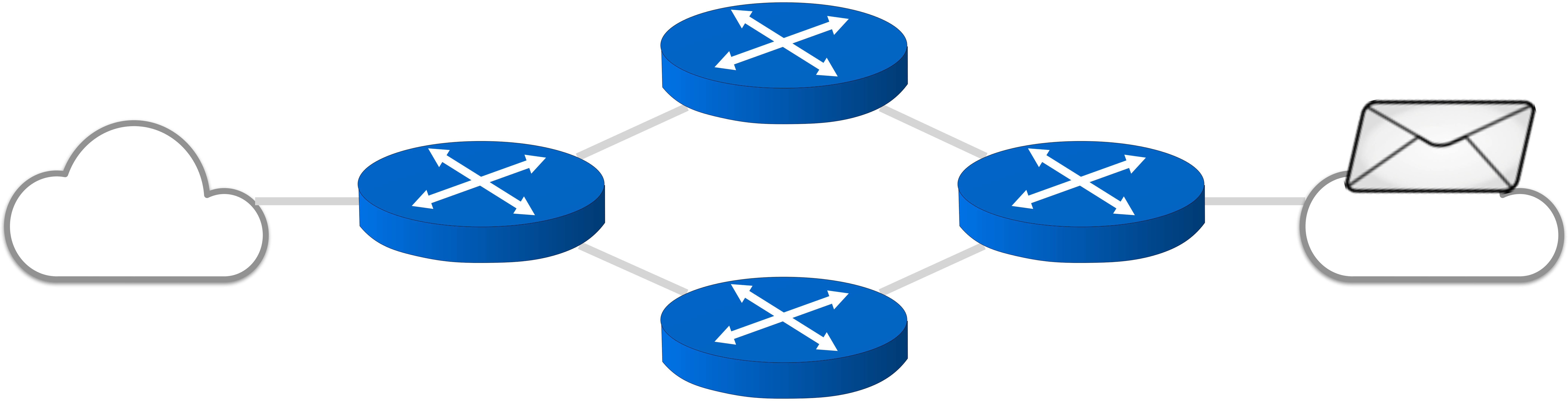
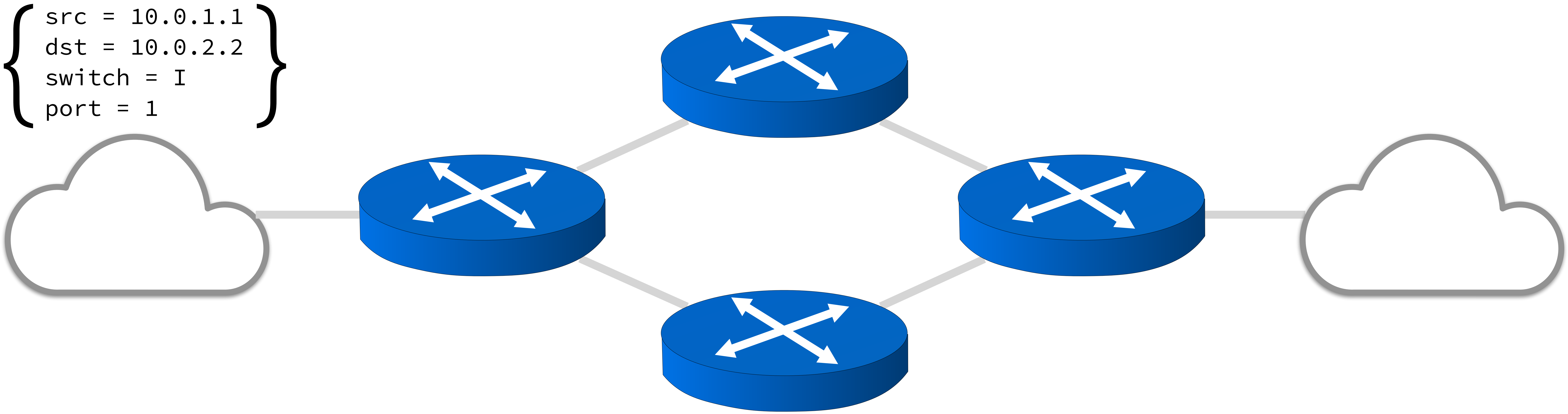
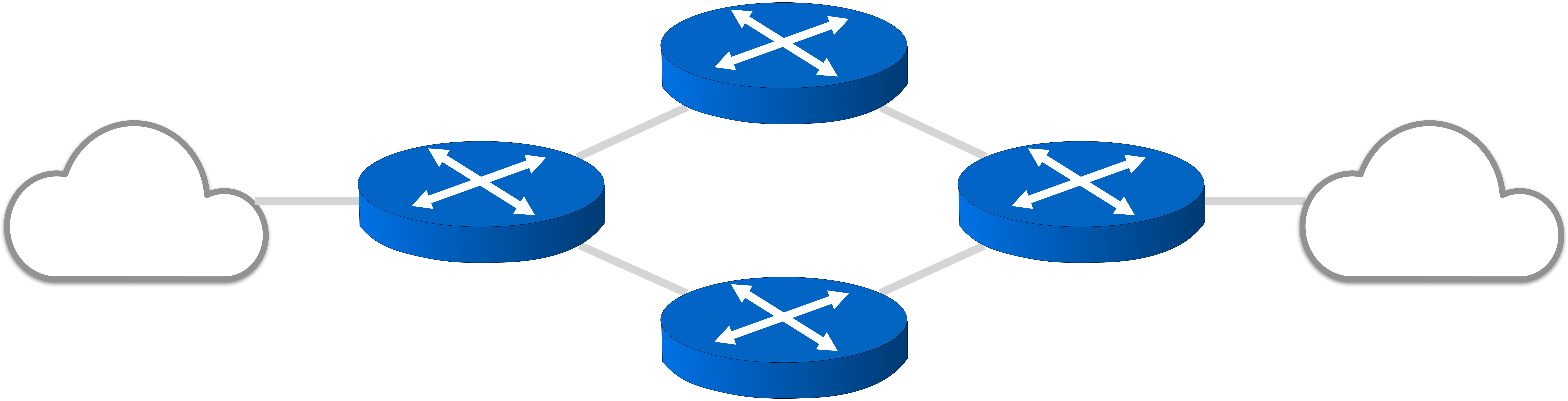


Packets: Records of fixed-width data



Packets: Records of fixed-width data

```
{
  src = 10.0.1.1
  dst = 10.0.2.2
  switch = A
  port = 2
}
```



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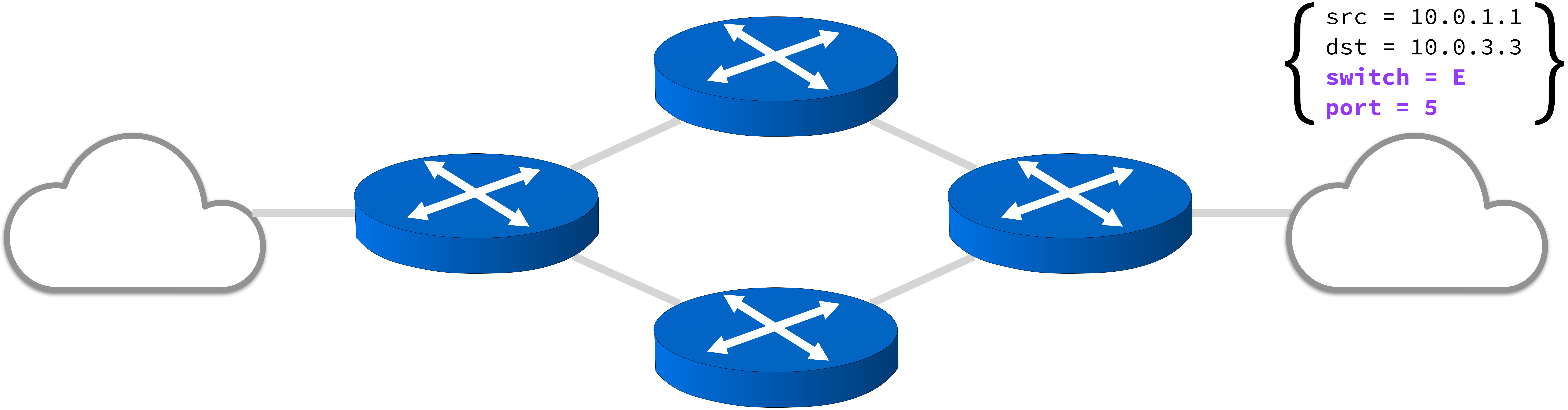
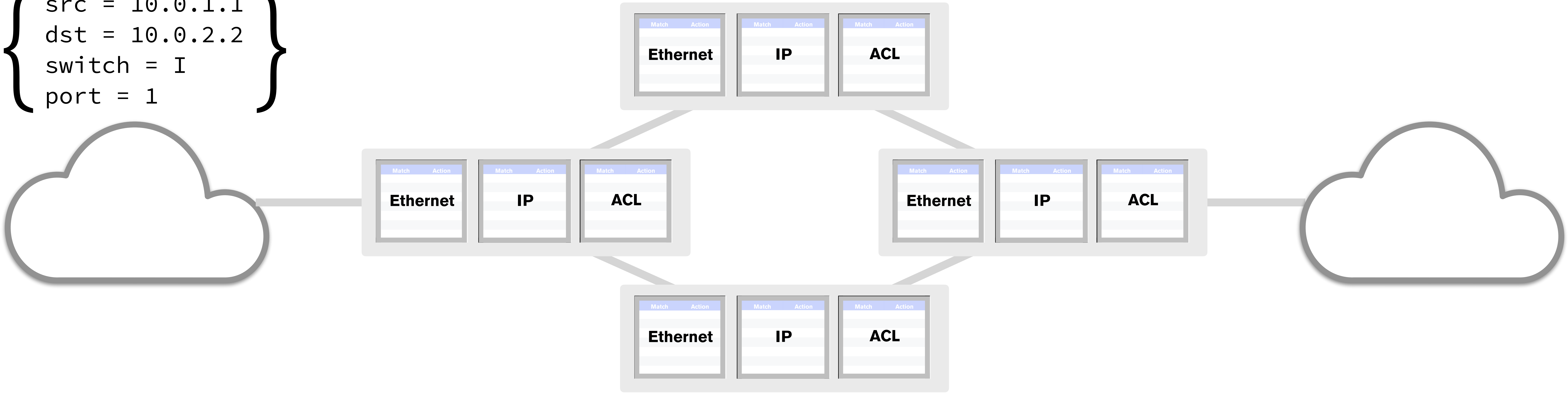


Network: Graphs of pipelines

```
src = 10.0.1.1
dst = 10.0.2.2
switch = I
port = 1
```



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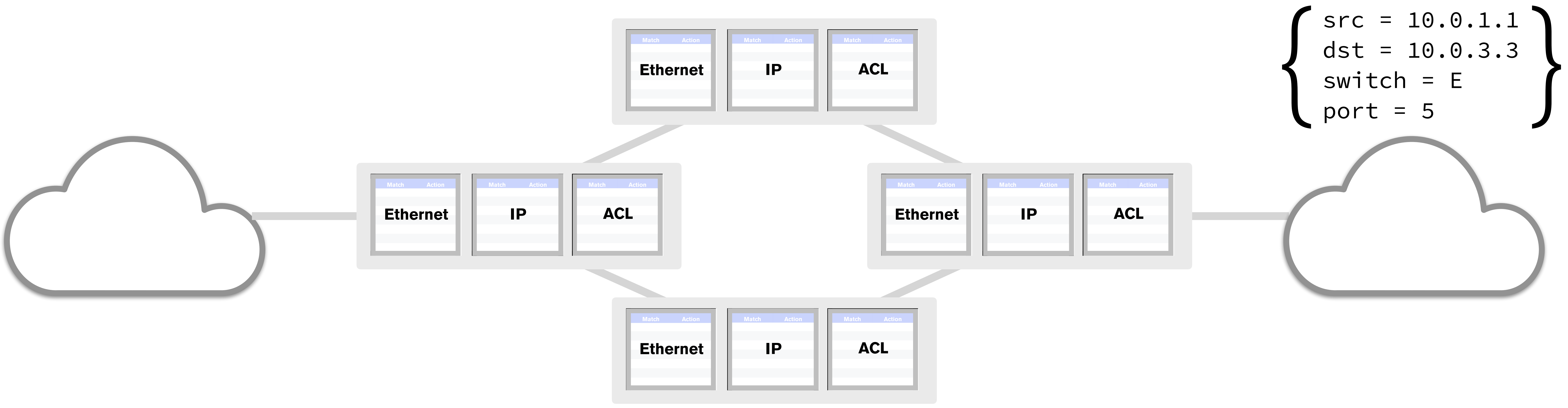

Design considerations

Packet Classification
To model match-action tables, need predicates evaluated packet headers (and other variables)

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To model richer pipelines, need operators that compose smaller programs both conditionally and in sequence

Iteration
Perhaps surprisingly, to model the potentially iterated processing performed via the topology, need general loops

NetKAT syntax

```
p, q ::=
  | id
  | drop
  | f = n
  | !p
  | f := n
  | p ; q
  | p + q
  | p*
  | dup
```

NetKAT predicates

```
p, q ::=
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  | f := n
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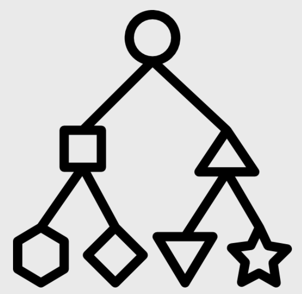

Packet Classification
To model match-action tables, need predicates evaluated packet headers (and other variables)

Match	Action
tcp_dst=22; ip_dst=10.0.1.1	allow
tcp_dst=22	deny
ip_dst=10.0.2.2	allow
ip_dst=10.0.3.3	allow
*	deny

NetKAT transformations

```
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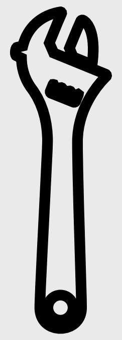

Transformations
To model behavior of switches, need imperative updates on packets and a way to delimit end of processing at a switch

```
A ⟹ B
 ≝
dup; sw = A; sw := B; dup
```

NetKAT composition operators

```
p, q ::=
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Modular Composition
To model richer pipelines, need operators that compose smaller programs both conditionally and in sequence

```
if p then q else r
        def
        ===
(p ; q) + (!p ; r)
```

NetKAT composition operators

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```

Note: “;” and “+” are overloaded as predicates

Modular Composition
To model richer pipelines, need operators that compose smaller programs both conditionally and in sequence

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(p ; q) + (!p ; r)
```

NetKAT iteration

```
p, q ::=
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  | f = n
  | !p
  | f := n
  | p ; q
  | p + q
  | p*
  | dup
```

Iteration
Perhaps surprisingly, to model the potentially iterated processing performed via the topology, need general loops

```
while p do q
```

$\overset{\text{def}}{=}$

```
(p ; q)* ; !p
```

NetKAT semantics

```
p, q ::=
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  | f = n
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```

Informal: NetKAT programs denote *functions* that take an input packet and produce a set of packet traces (i.e., non-empty lists).

Formal: $[\![p]\!] \in \text{Packet} \rightarrow \mathcal{P}(\text{Packet}^{+})$

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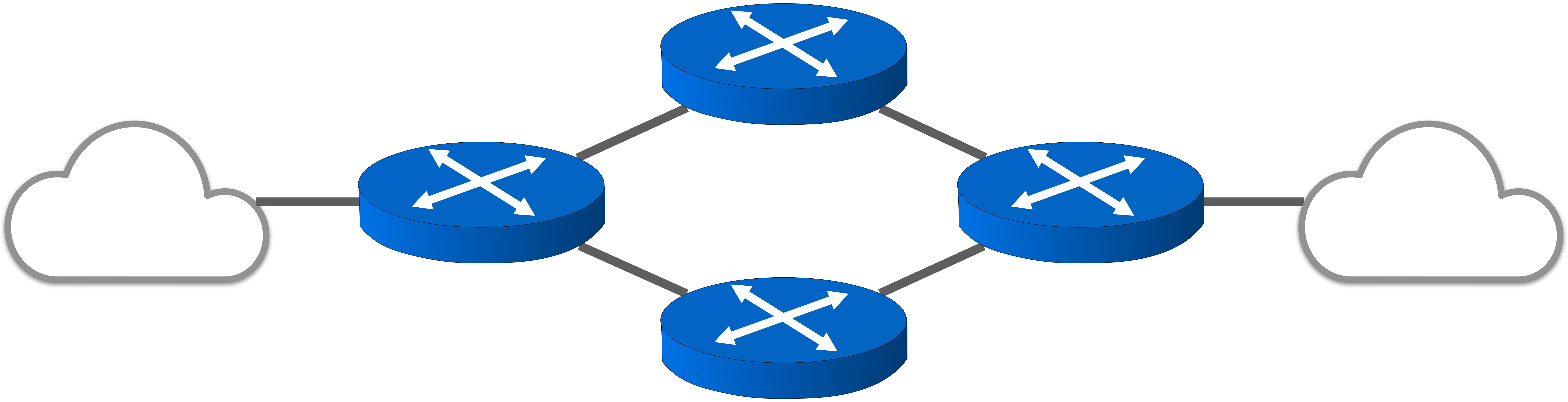
FAQ

Q: *Why a set?*
A: Enables dropping packets + multicast

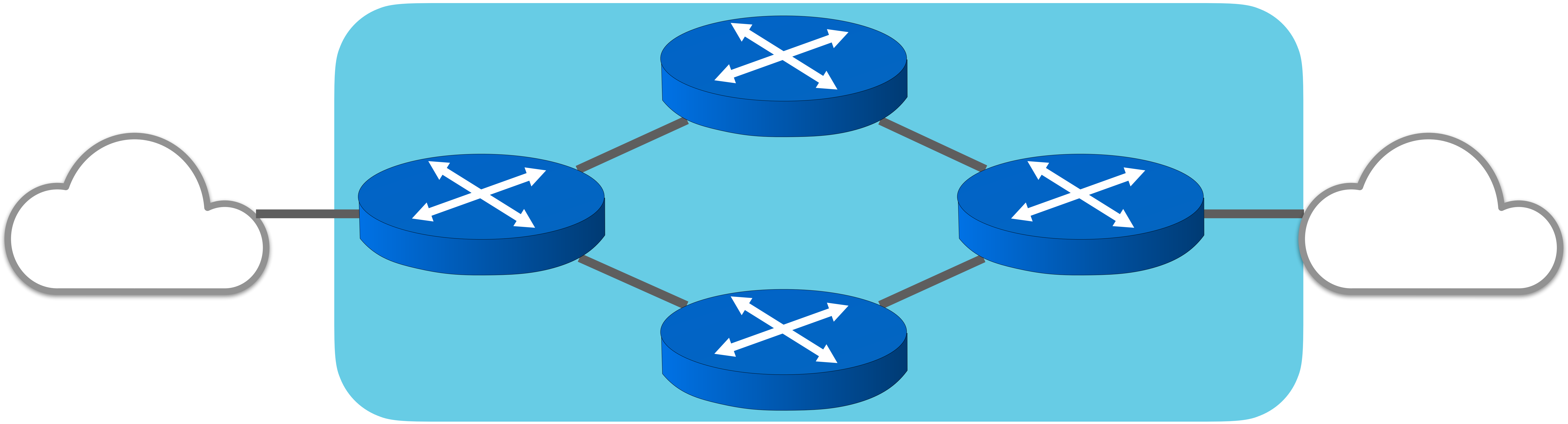
Q: *Why a trace?*
A: Captures end-to-end forwarding path

Q: *Why a function?*
A: Simple model of "mostly stateless" forwarding

Modeling networks in NetKAT



Modeling networks in NetKAT



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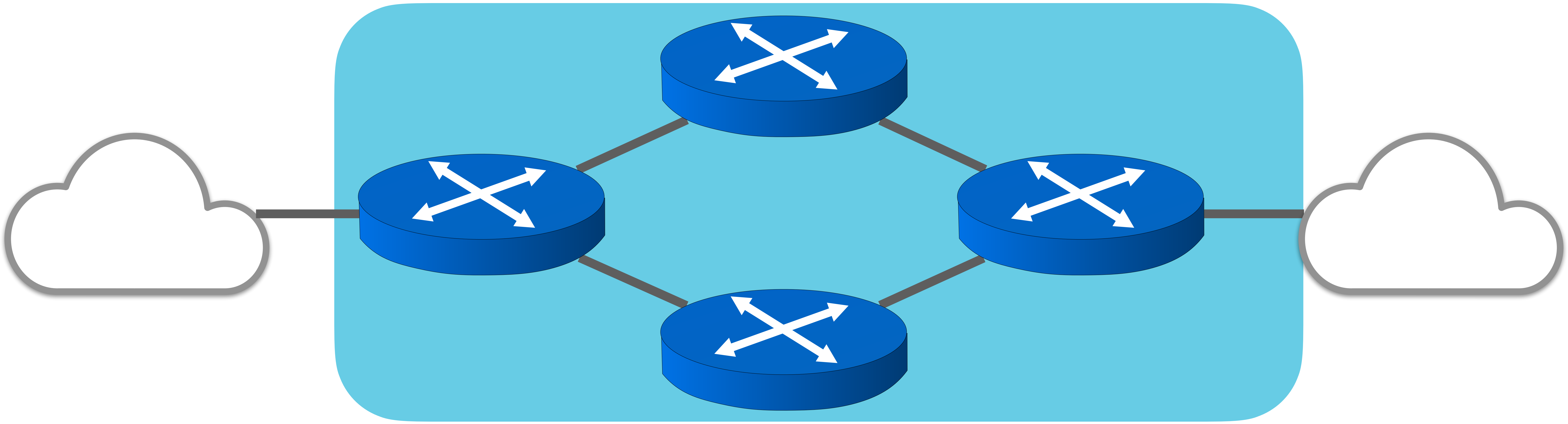
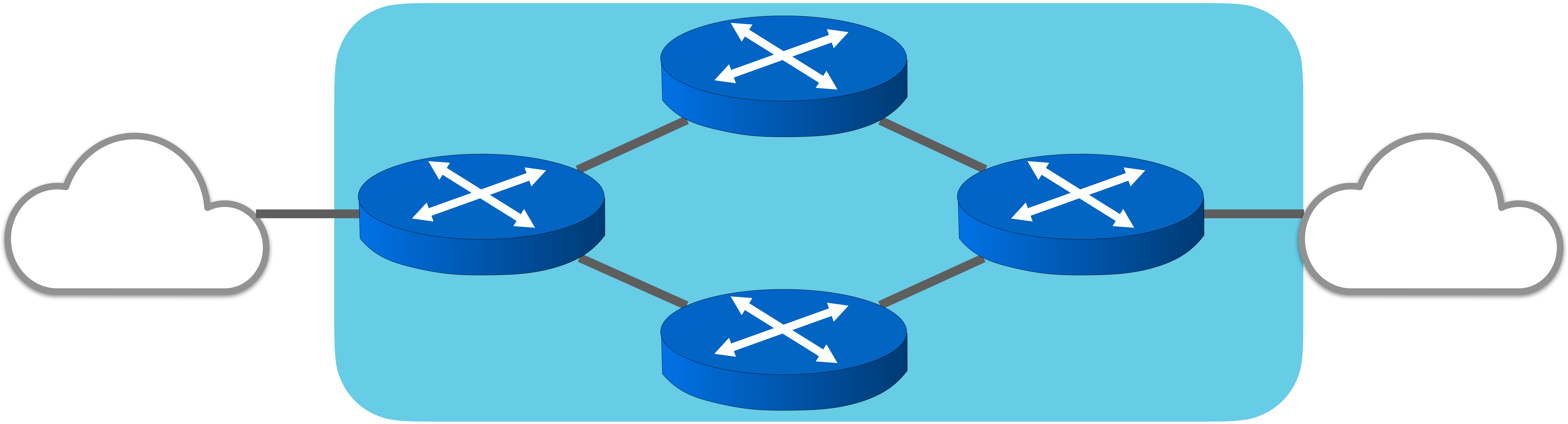


switch
Models the processing done at each switch

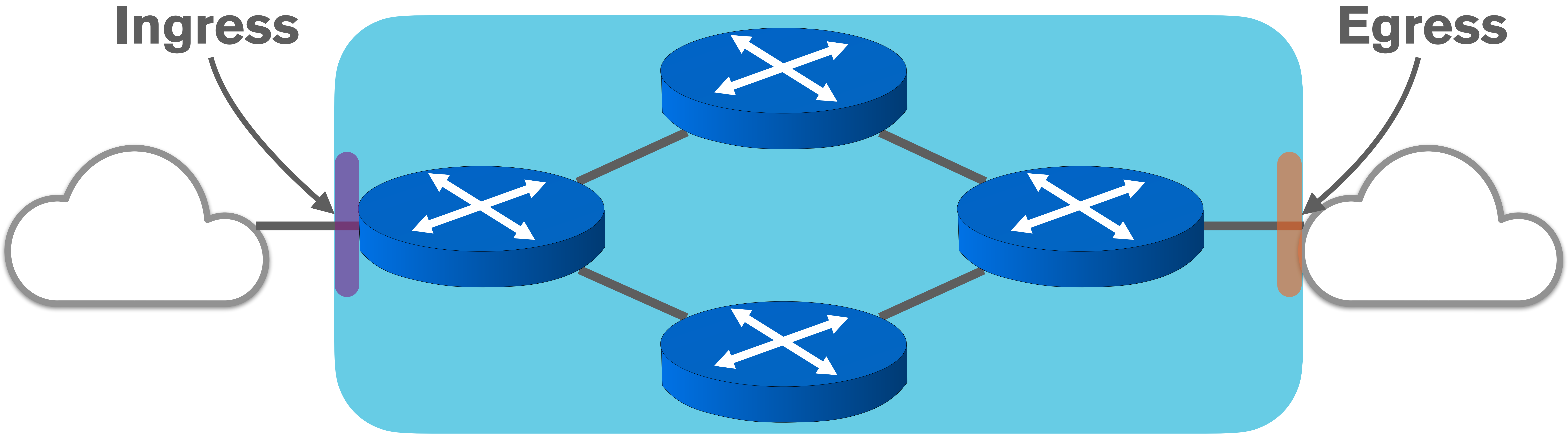
Modeling networks in NetKAT



switch
Models the processing done at each switch

topology
Models the forwarding done by each link

Modeling networks in NetKAT

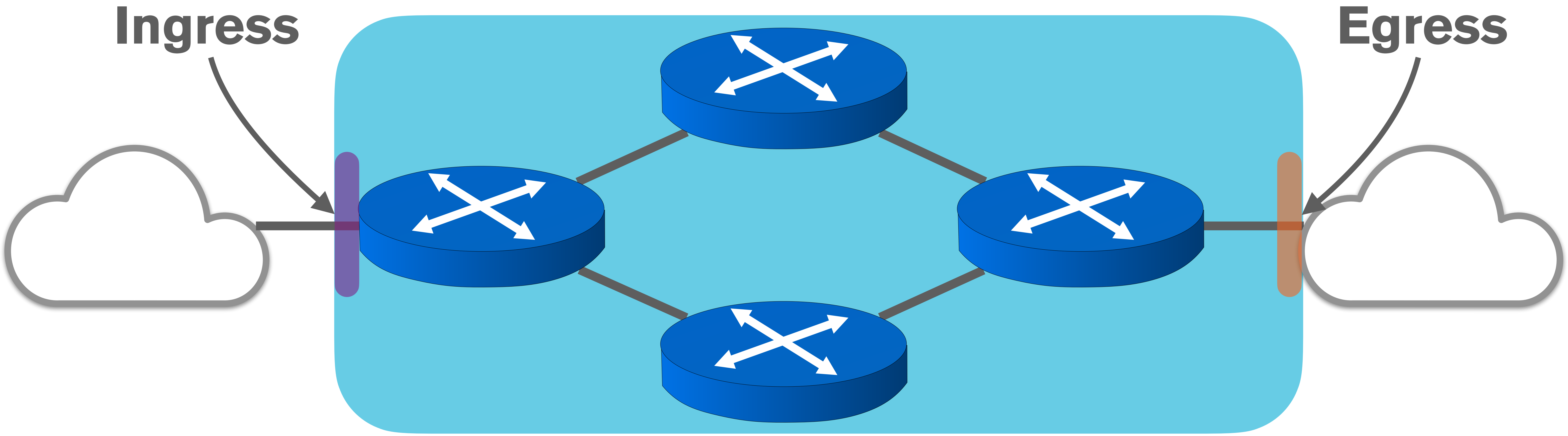


switch
Models the processing done at each switch

topology
Models the forwarding done by each link

ingress & egress
Models the perimeter of the network

Modeling networks in NetKAT



switch
Models the processing done at each switch

topology
Models the forwarding done by each link

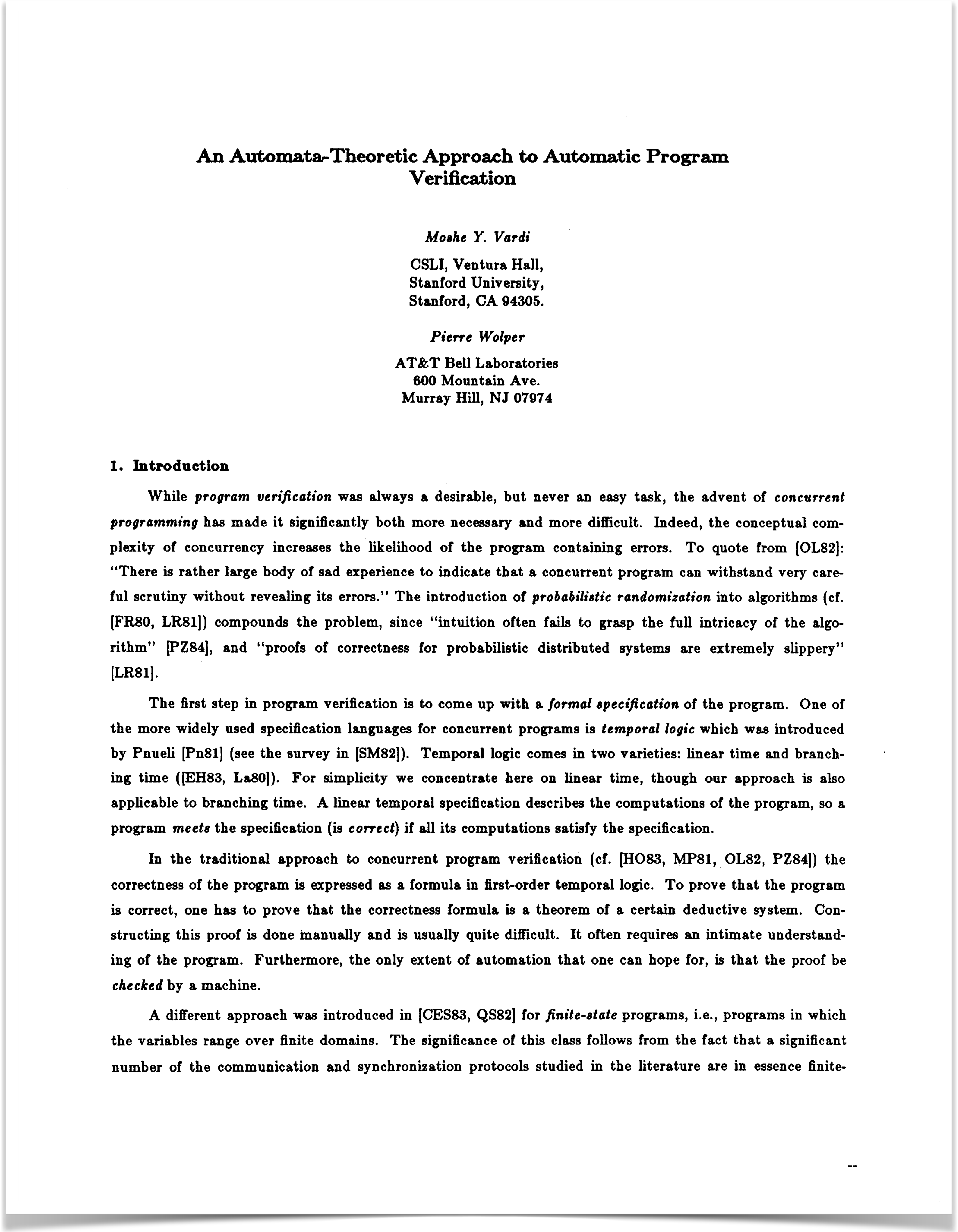
ingress & egress
Models the perimeter of the network

```
ingress ; (switch ; topology)* ; egress
```

Verification via equivalence

Idea: encode the program and its specification in a unified framework, then check equivalence (or inclusion, etc.)

History: A classic approach, pioneered by Vardi & Wolper, and widely used in hardware and software verification


An Automata-Theoretic Approach to Automatic Program Verification

Moshe Y. Vardi

CSLI, Ventura Hall,
Stanford University,
Stanford, CA 94305.

Pierre Wolper

AT&T Bell Laboratories
600 Mountain Ave.
Murray Hill, NJ 07974

1. Introduction

While *program verification* was always a desirable, but never an easy task, the advent of *concurrent programming* has made it significantly both more necessary and more difficult. Indeed, the conceptual complexity of concurrency increases the likelihood of the program containing errors. To quote from [OL82]: "There is rather large body of sad experience to indicate that a concurrent program can withstand very careful scrutiny without revealing its errors." The introduction of *probabilistic randomization* into algorithms (cf. [FR80, LR81]) compounds the problem, since "intuition often fails to grasp the full intricacy of the algorithm" [PZ84], and "proofs of correctness for probabilistic distributed systems are extremely slippery" [LR81].

The first step in program verification is to come up with a *formal specification* of the program. One of the more widely used specification languages for concurrent programs is *temporal logic* which was introduced by Pnueli [Pn81] (see the survey in [SM82]). Temporal logic comes in two varieties: linear time and branching time ([EH83, La80]). For simplicity we concentrate here on linear time, though our approach is also applicable to branching time. A linear temporal specification describes the computations of the program, so a program *meets* the specification (is *correct*) if all its computations satisfy the specification.

In the traditional approach to concurrent program verification (cf. [HO83, MP81, OL82, PZ84]) the correctness of the program is expressed as a formula in first-order temporal logic. To prove that the program is correct, one has to prove that the correctness formula is a theorem of a certain deductive system. Constructing this proof is done manually and is usually quite difficult. It often requires an intimate understanding of the program. Furthermore, the only extent of automation that one can hope for, is that the proof be *checked* by a machine.

A different approach was introduced in [CES83, QS82] for *finite-state* programs, i.e., programs in which the variables range over finite domains. The significance of this class follows from the fact that a significant number of the communication and synchronization protocols studied in the literature are in essence finite-

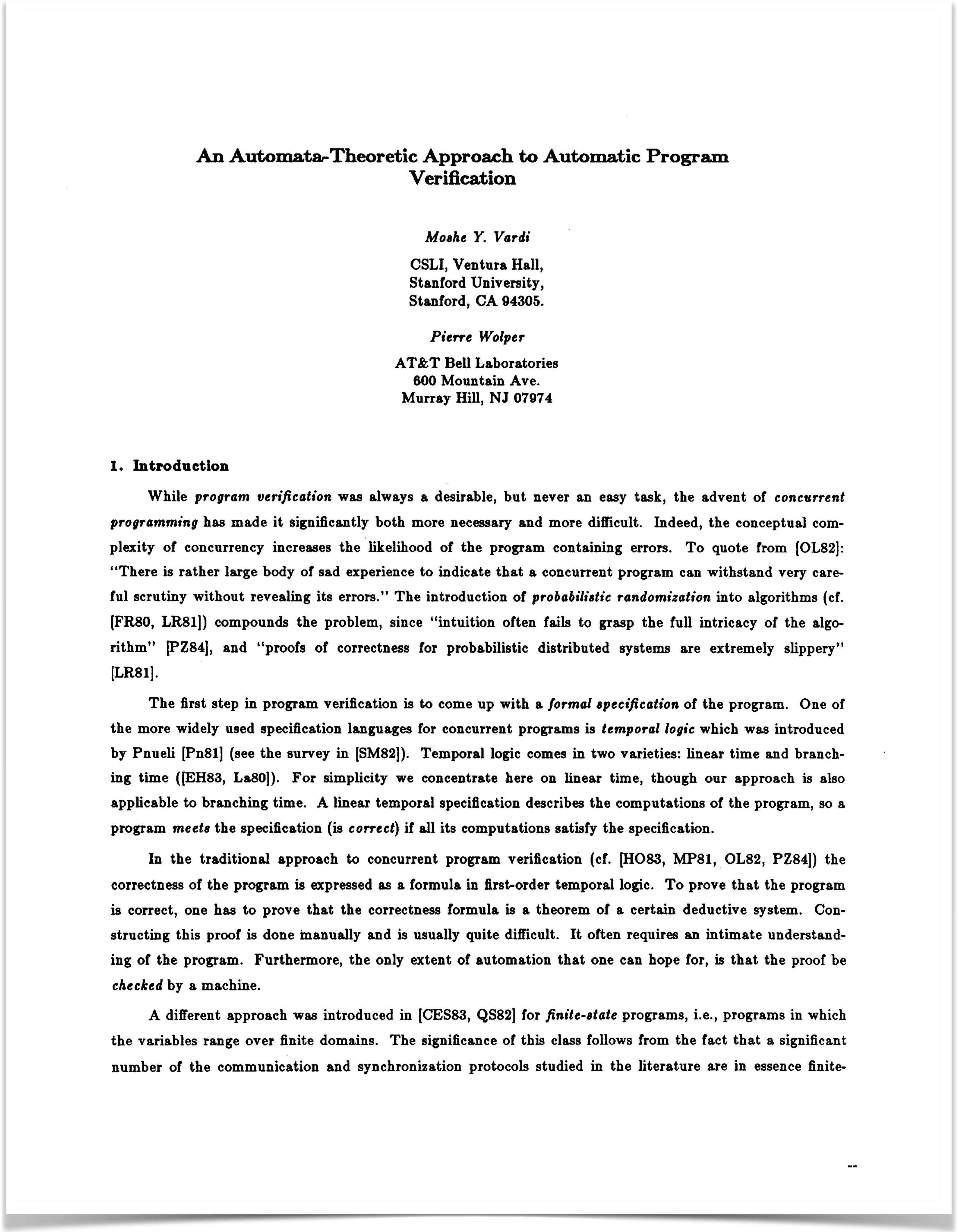
Verification via equivalence

Idea: encode the program and its specification in a unified framework, then check equivalence (or inclusion, etc.)

History: A classic approach, pioneered by Vardi & Wolper, and widely used in hardware and software verification

Questions:

1. Can NetKAT encode useful specifications?
2. Is program equivalence decidable (and if so, can we come up with practical approaches for checking it?)



An Automata-Theoretic Approach to Automatic Program Verification

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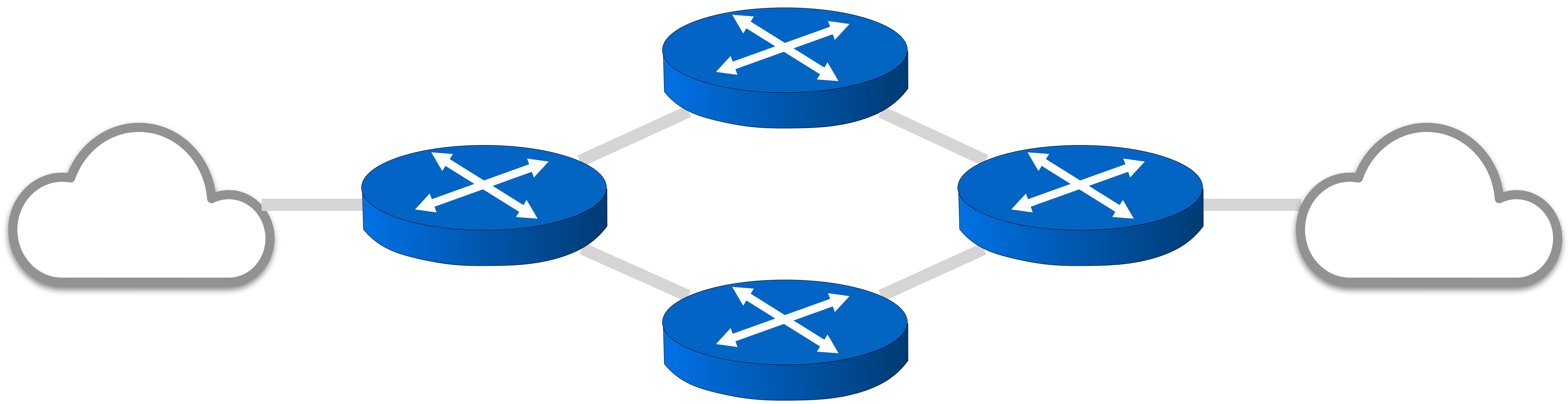
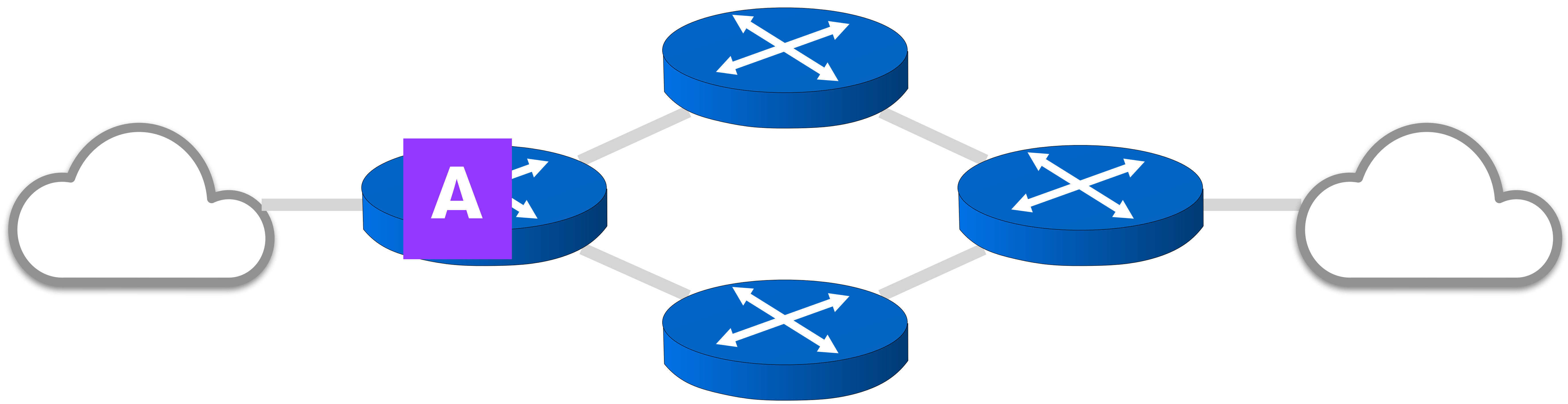
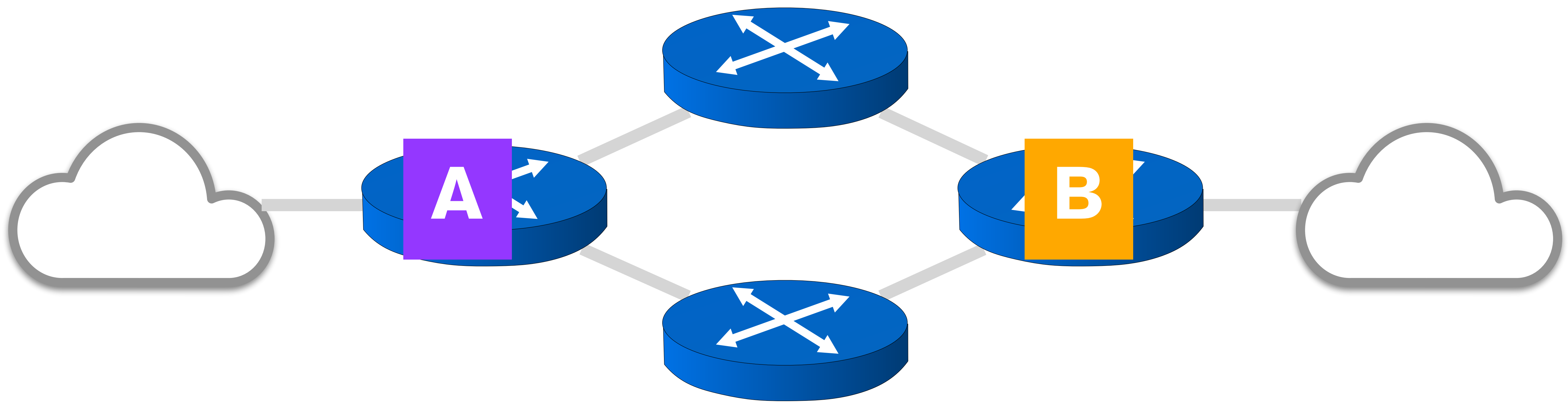
Example: reachability



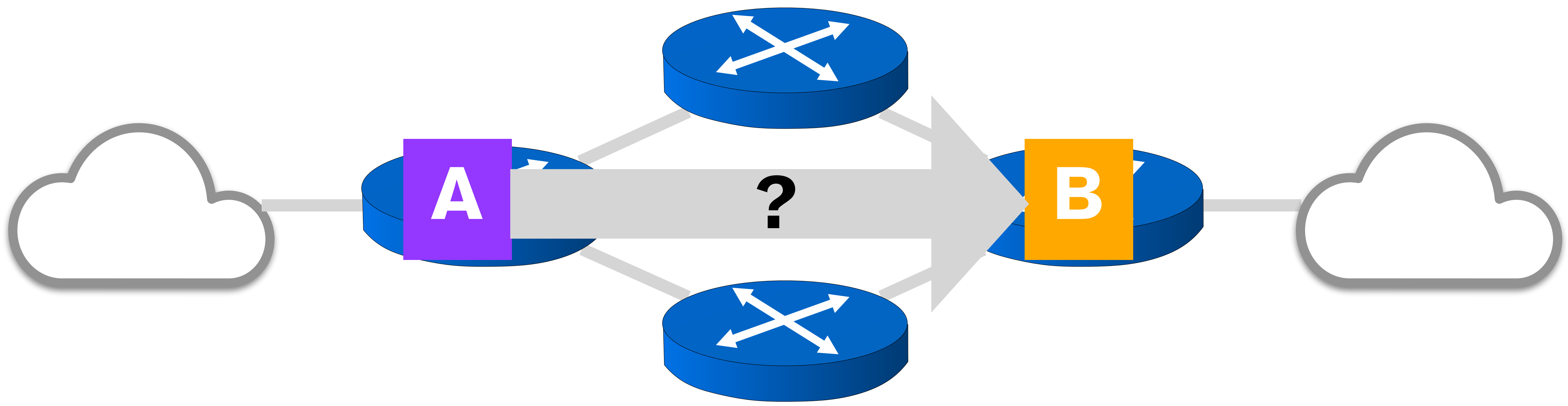
Example: reachability



Example: reachability



Example: reachability



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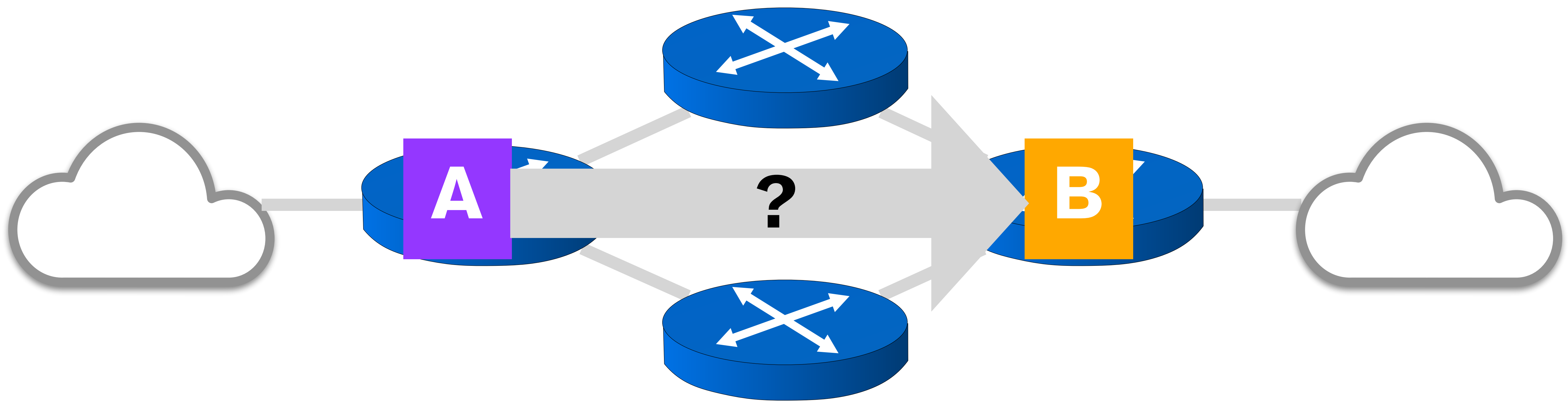
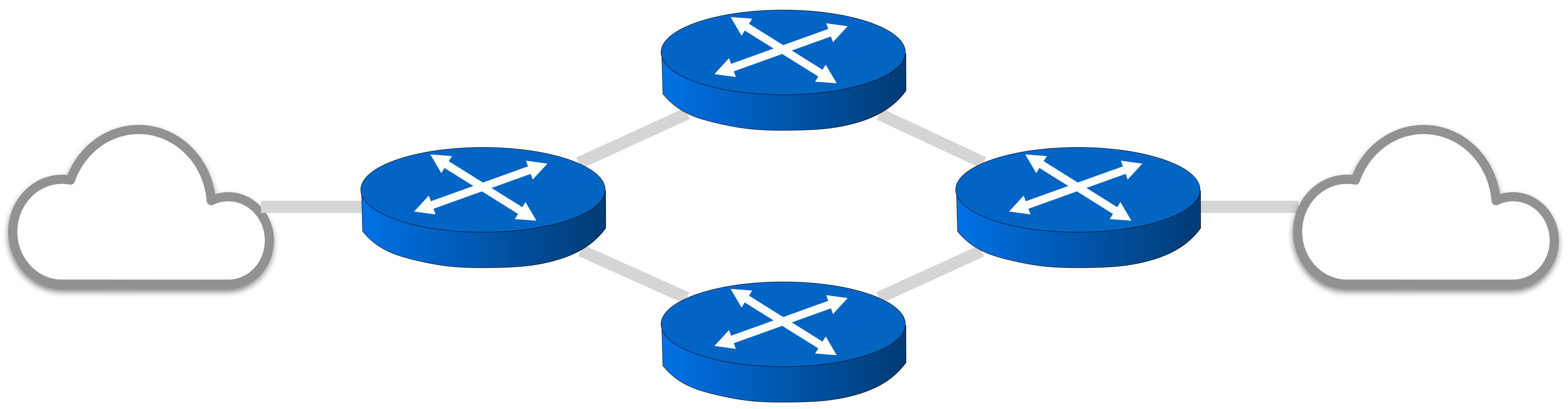


Property: B reachable from A

Approach:
- Build a model with A as ingress and B as egress
- Check for non-emptiness

Query: `switch=A; (switch; topo)*; switch=B ≢ drop`

Example: isolation



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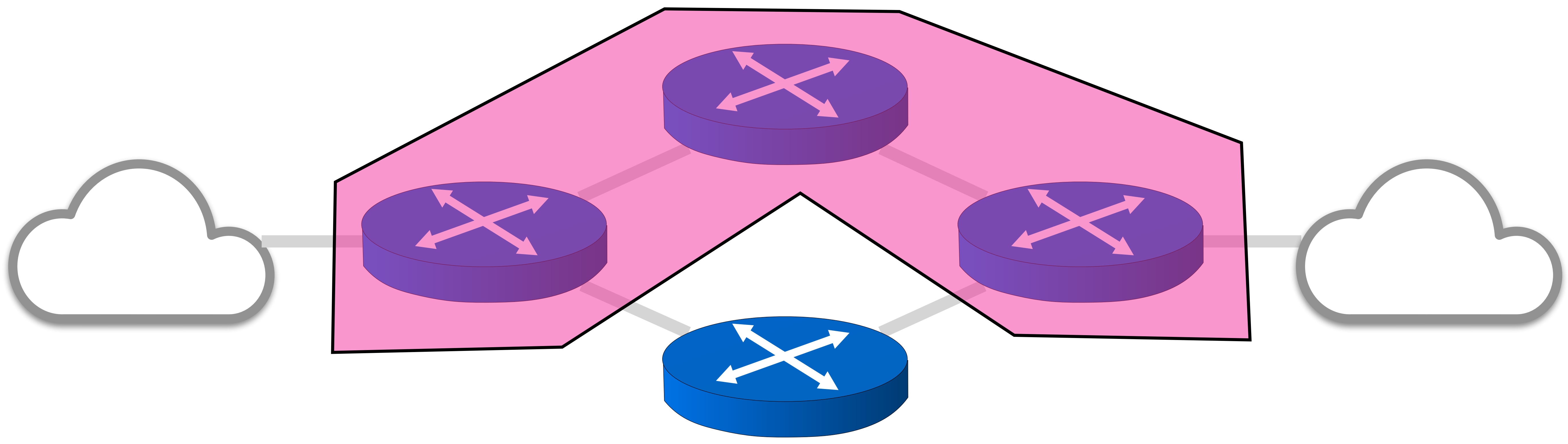

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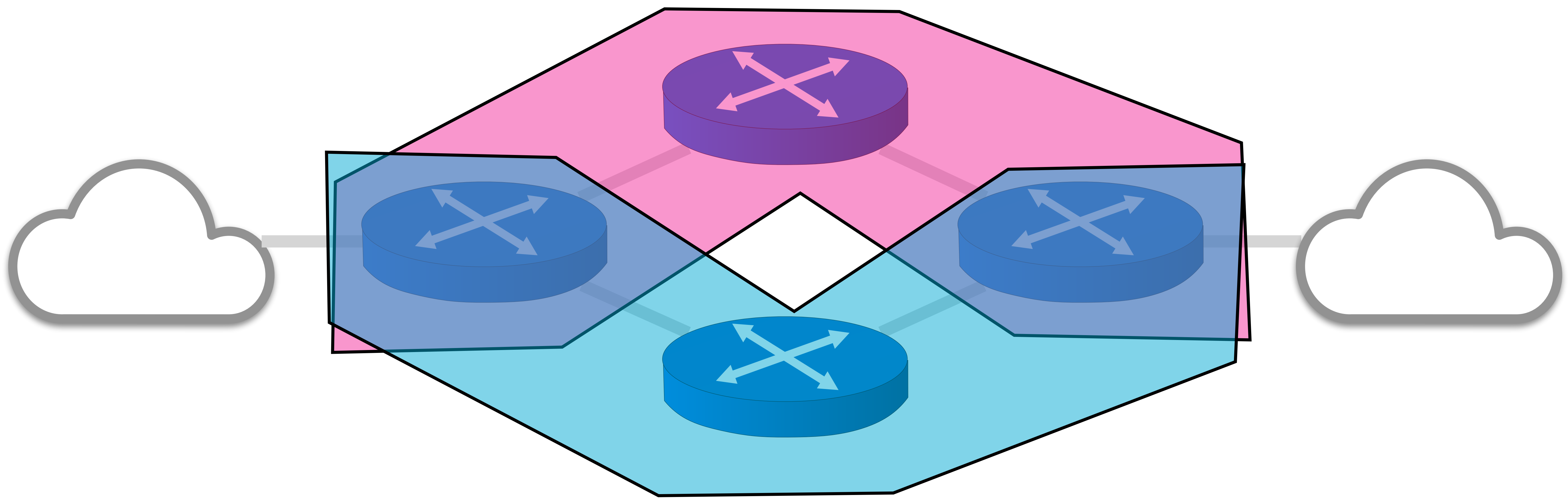
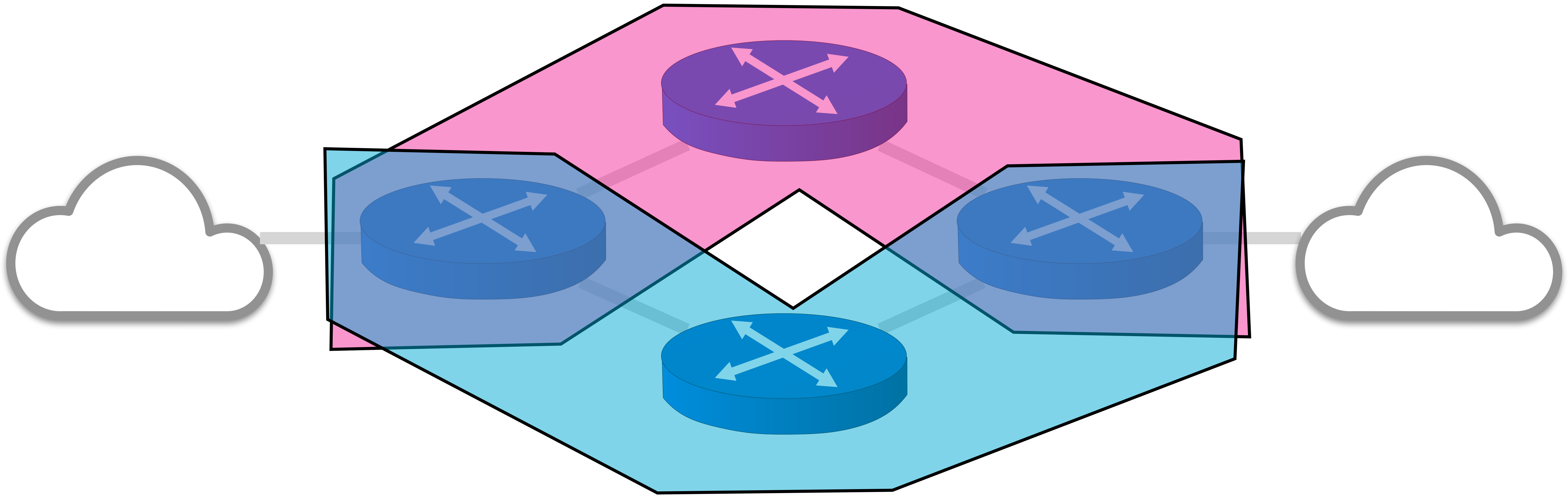


Example: isolation



Property: slice s logically isolated from q

Approach: Check that running s and q together is equivalent to running them independently on separate “copies” of the network

Query: `in; ([s + q] ; topo)*; eg ≡`

`[in; (s ; topo)*; eg] + [in; (q ; topo)*; eg]`

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Categories and Subject Descriptors F.3.1 [*Specifying and Verifying and Reasoning about Programs*]: Mechanical verification

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These tools represent a good first step toward making networks more reliable, but they have two important limitations. First, they are based on ad hoc foundations. Although SDN platforms such as OpenFlow [21] have precise (if informal) specifications, the tools make simplifying assumptions that are routinely violated by real

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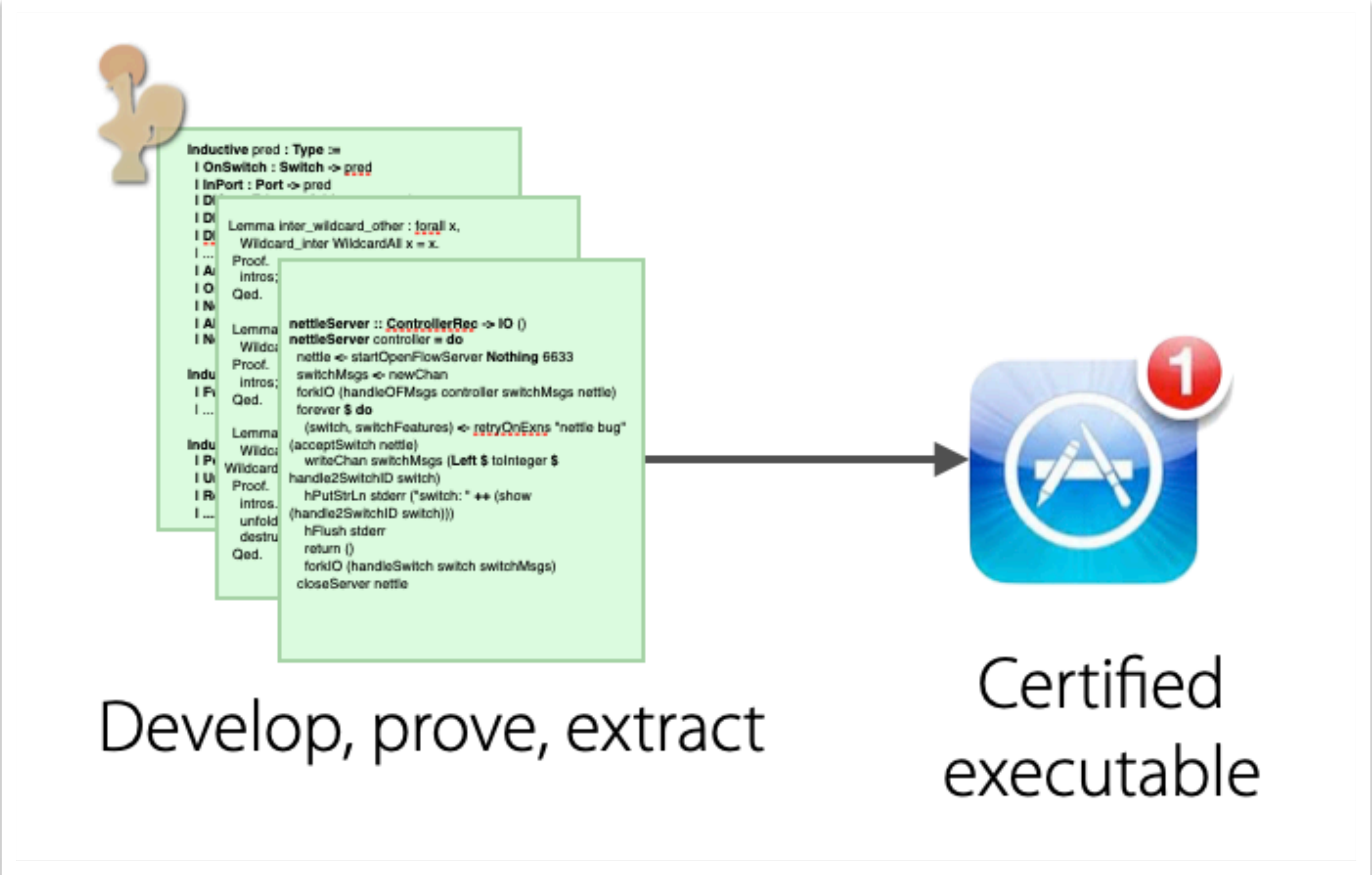

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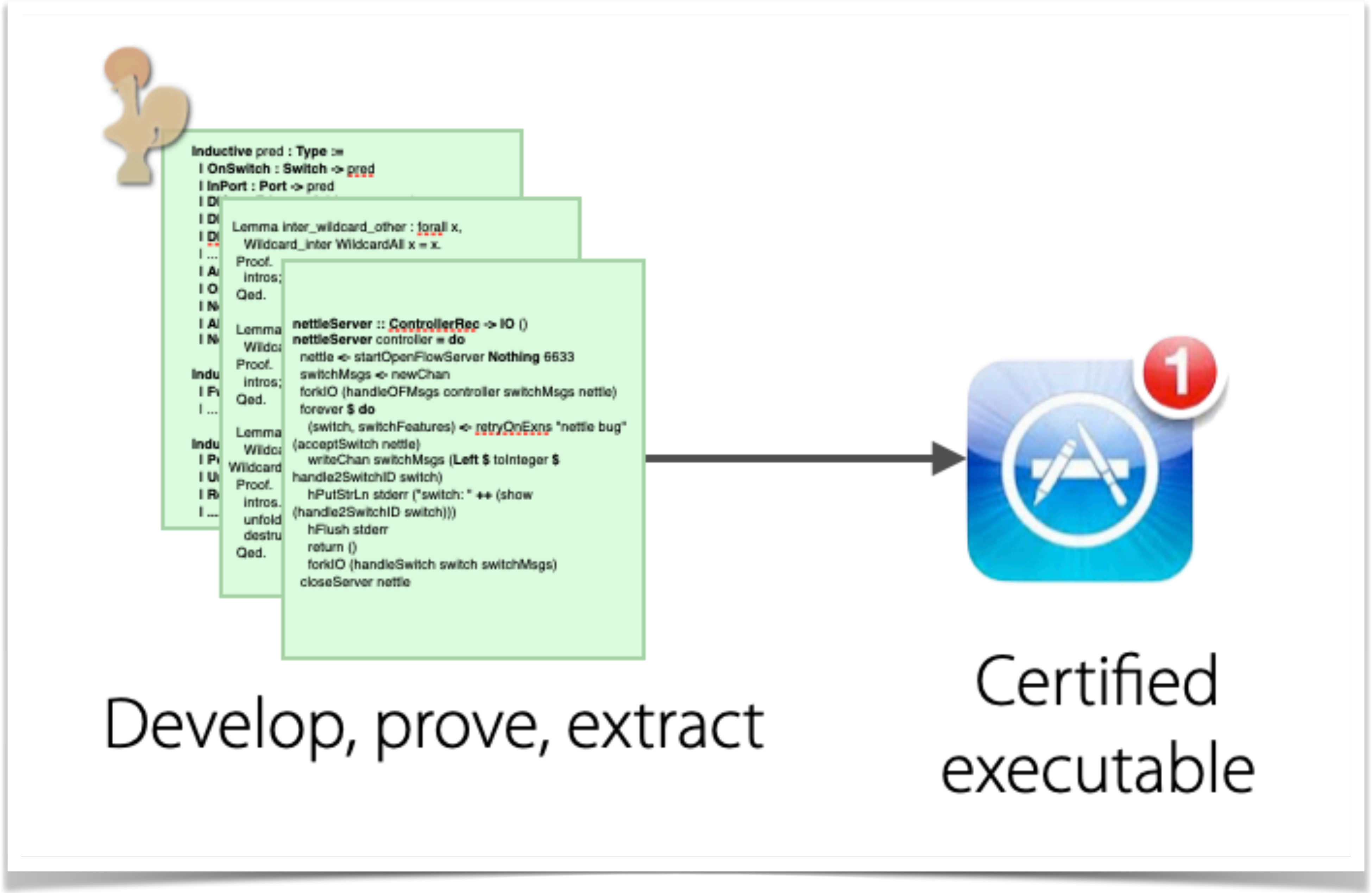

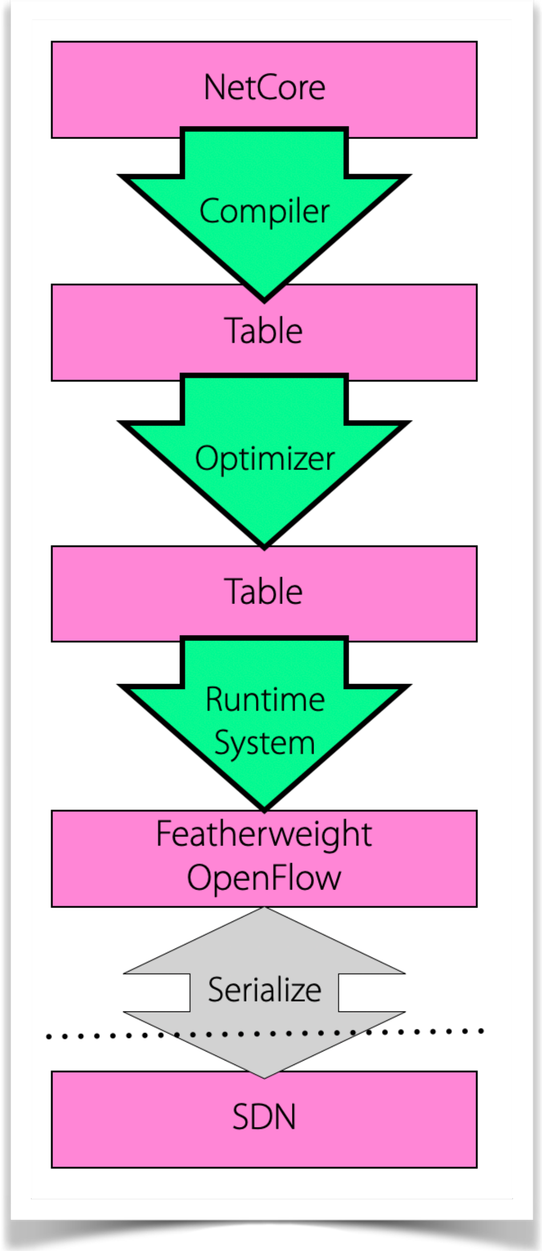

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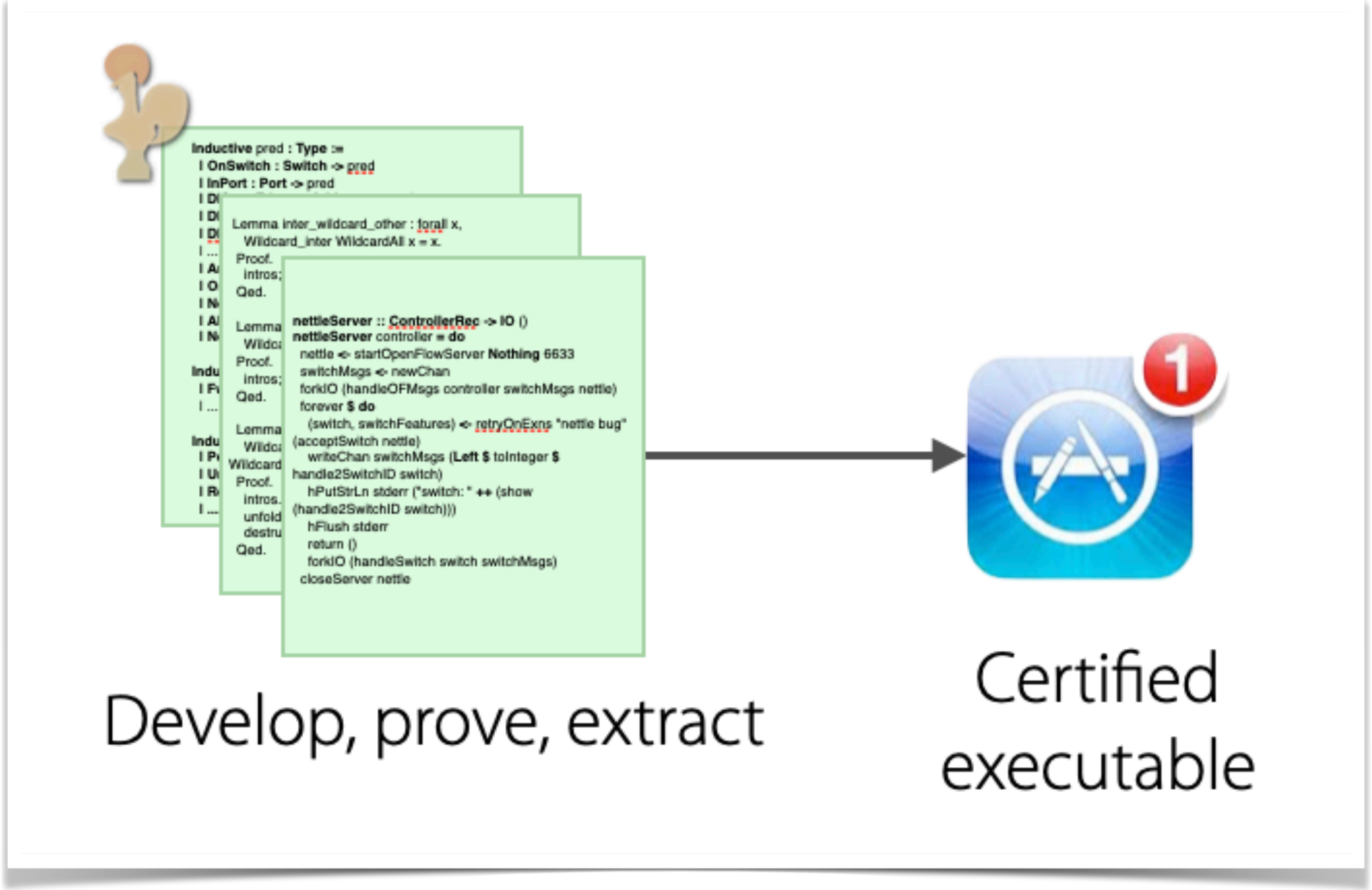
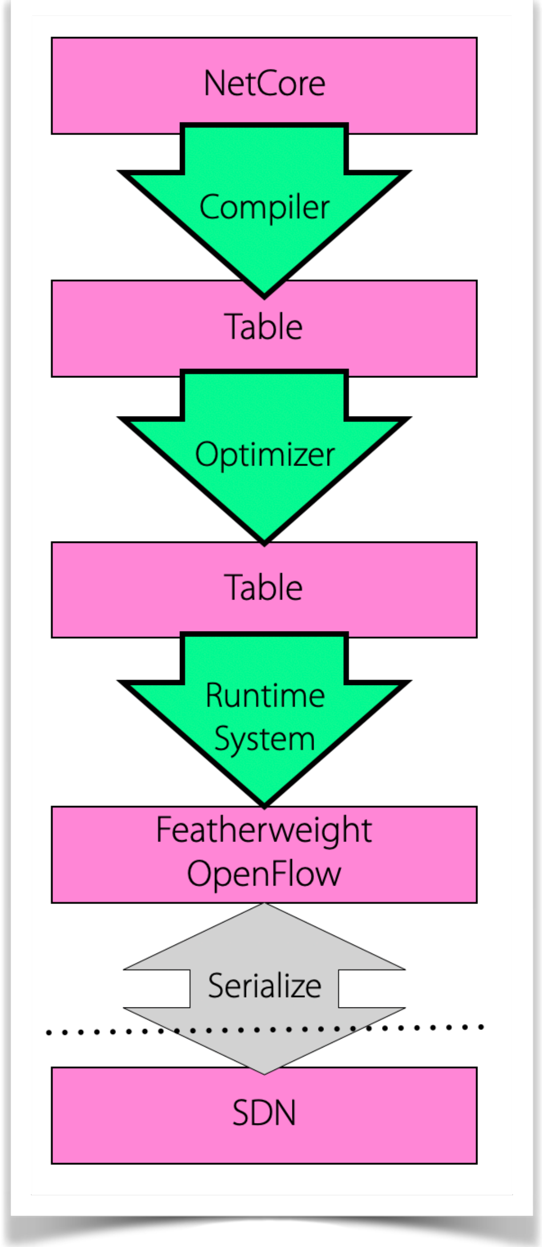
Ugh, I'm *sick* of re-doing
this compiler proof! But I model the
semantics as a semi-ring, I can factor out
most of it out!





Equational Axioms

Kleene Algebra Axioms [Kozen ’94]

p + (q + r) ≡ (p + q) + r
p + q ≡ q + p
p + drop ≡ p
p + p ≡ p
p; (q; r) ≡ (p; q); r
p; (q + r) ≡ p; q + p; r
(p + q); r ≡ p; r + q; r
id; p ≡ p
p ≡ p; id
drop; p ≡ drop
p; drop ≡ drop
id + p; p* ≡ p*
id + p*; p ≡ p*
p + q; r + r ≡ r ⇒ p*; q + r ≡ r
p + q; r + q ≡ q ⇒ p; r* + q ≡ q

Additional Boolean Algebra Axioms

a + (b; c) ≡ (a + b); (a + c)
a + id ≡ id
a + ! a ≡ id
a; b ≡ b; a
a; !a ≡ drop
a; a ≡ a

Packet Axioms (for f≠g, n≠m)

f := n; g := m ≡ g := m; f := n
f := n; g = m ≡ g = m; f := n
f := n; f = n ≡ f := n
f = n; f := n ≡ f = n
f := n; f := m ≡ f := m
f = n; f = m ≡ false
dup; f = n ≡ f = n; dup
Σ_i f = n_i ≡ true

Equational Axioms

Kleene Algebra Axioms [Kozen ’94]

p + (q + r) ≡ (p + q) + r
p + q ≡ q + p
p + drop ≡ p
p + p ≡ p
p; (q
p; (q
(p +
id; p
p ≡ p
drop; p ≡ drop
p; drop ≡ drop
id + p; p* ≡ p*
id + p*; p ≡ p*
p + q; r + r ≡ r ⇒ p*; q + r ≡ r
p + q; r + q ≡ q ⇒ p; r* + q ≡ q

Additional Boolean Algebra Axioms

a + (b; c) ≡ (a + b); (a + c)
a + id ≡ id
a + ! a ≡ id
a; b ≡ b; a

n
f := n; f = n ≡ f := n
f = n; f := n ≡ f = n
f := n; f := m ≡ f := m
f = n; f = m ≡ false
dup; f = n ≡ f = n; dup
Σ_i f = n_i ≡ true

Soundness: If ⊢ p ≡ q, then ⟦p⟧ = ⟦q⟧

Completeness: If ⟦p⟧ = ⟦q⟧, then ⊢ p ≡ q

NetKAT automata

We can also build automata that recognize packet traces

A *NetKAT Automaton* is a tuple $M=\langle S, s_0, \varepsilon, \delta \rangle$ where:

- S is a finite set of states,
- $s_0 \in S$ is the start state,
- $\varepsilon \in S \to \text{Packet} \to \text{Packet Set}$
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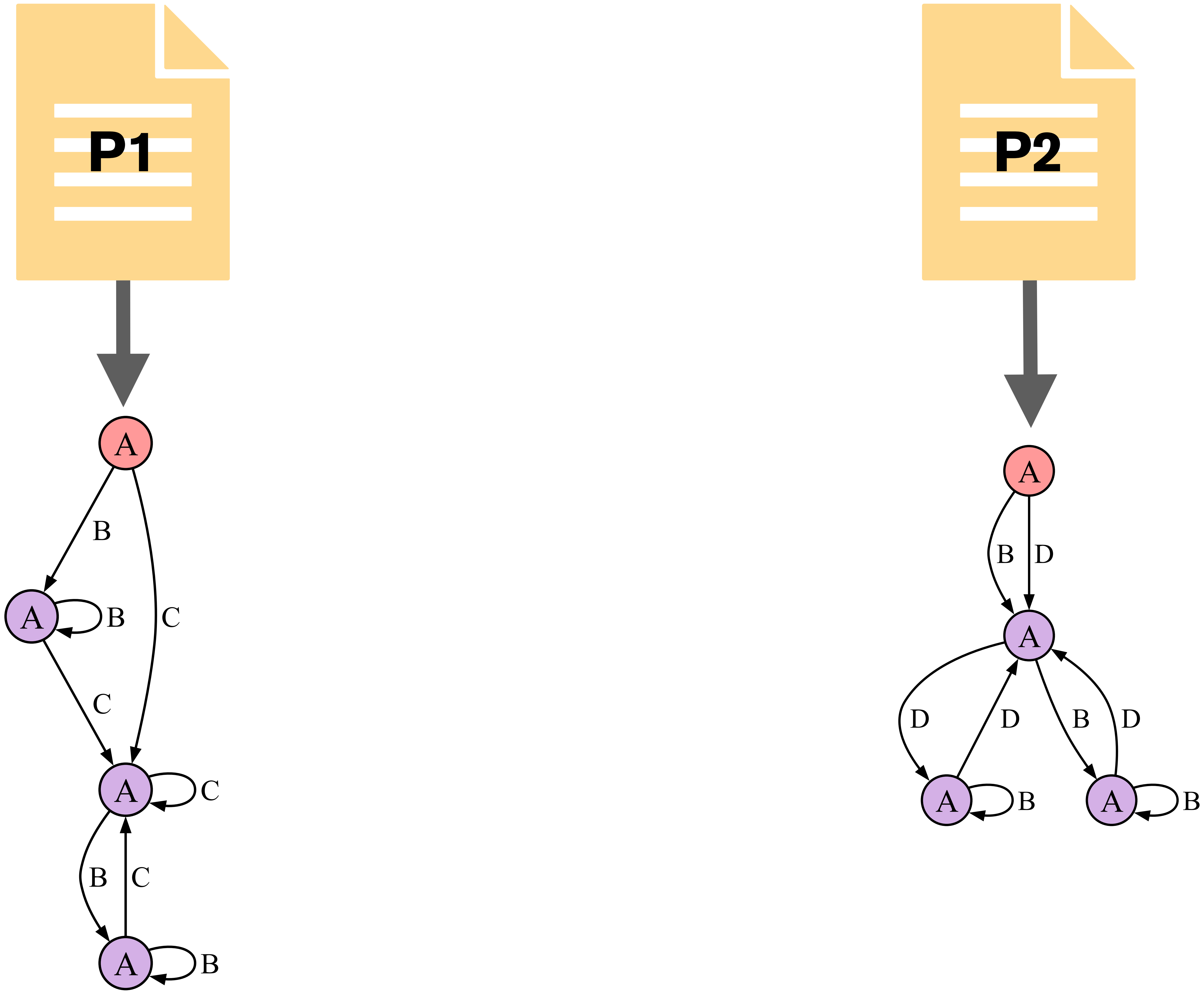
M *accepts* a trace in state s if:

- accept s [p, p'] $\Leftrightarrow$ $p' \in \varepsilon\ s\ p$
- accept s [p, p'] @ rest $\Leftrightarrow$ $\exists\, s'.\ (p', s') \in \delta\ s\ p \wedge$ accept s' (p' @ rest)

Checking equivalence



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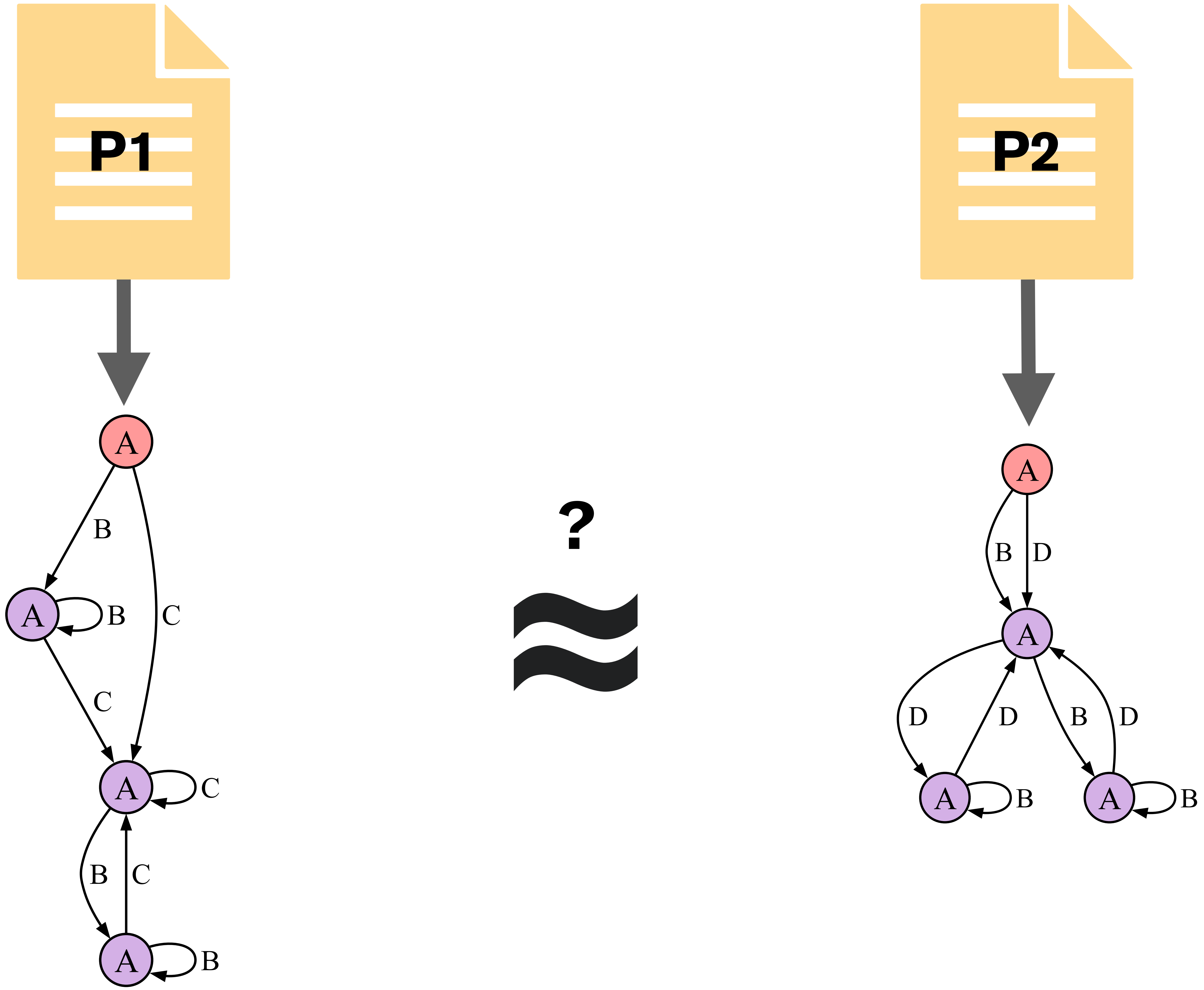


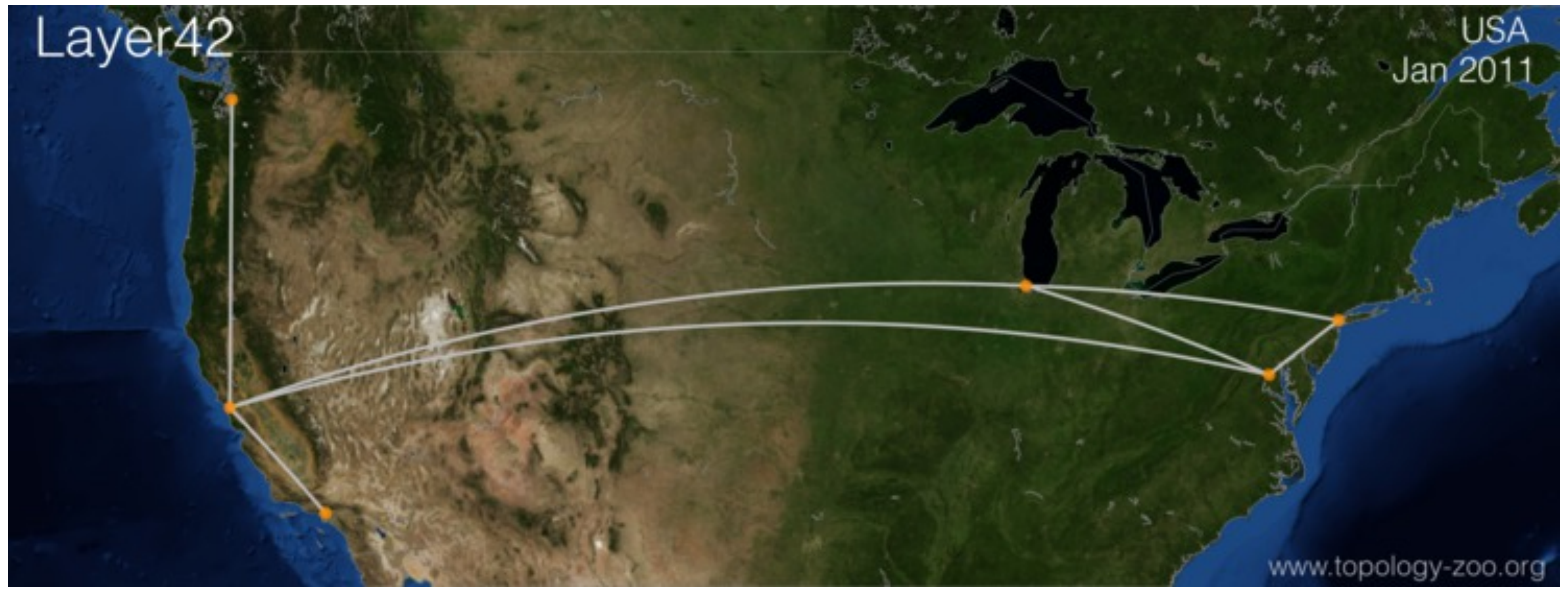
Evaluation: benchmark suite

Dataset
Internet Topology Zoo, a dataset of 140 real-world topologies, mostly large ISPs

Configurations
Synthetic programs that forward traffic along shortest paths

Property
All-pairs reachability

Key question
Performance relative to APKeep, a state-of-the-art network verification tool



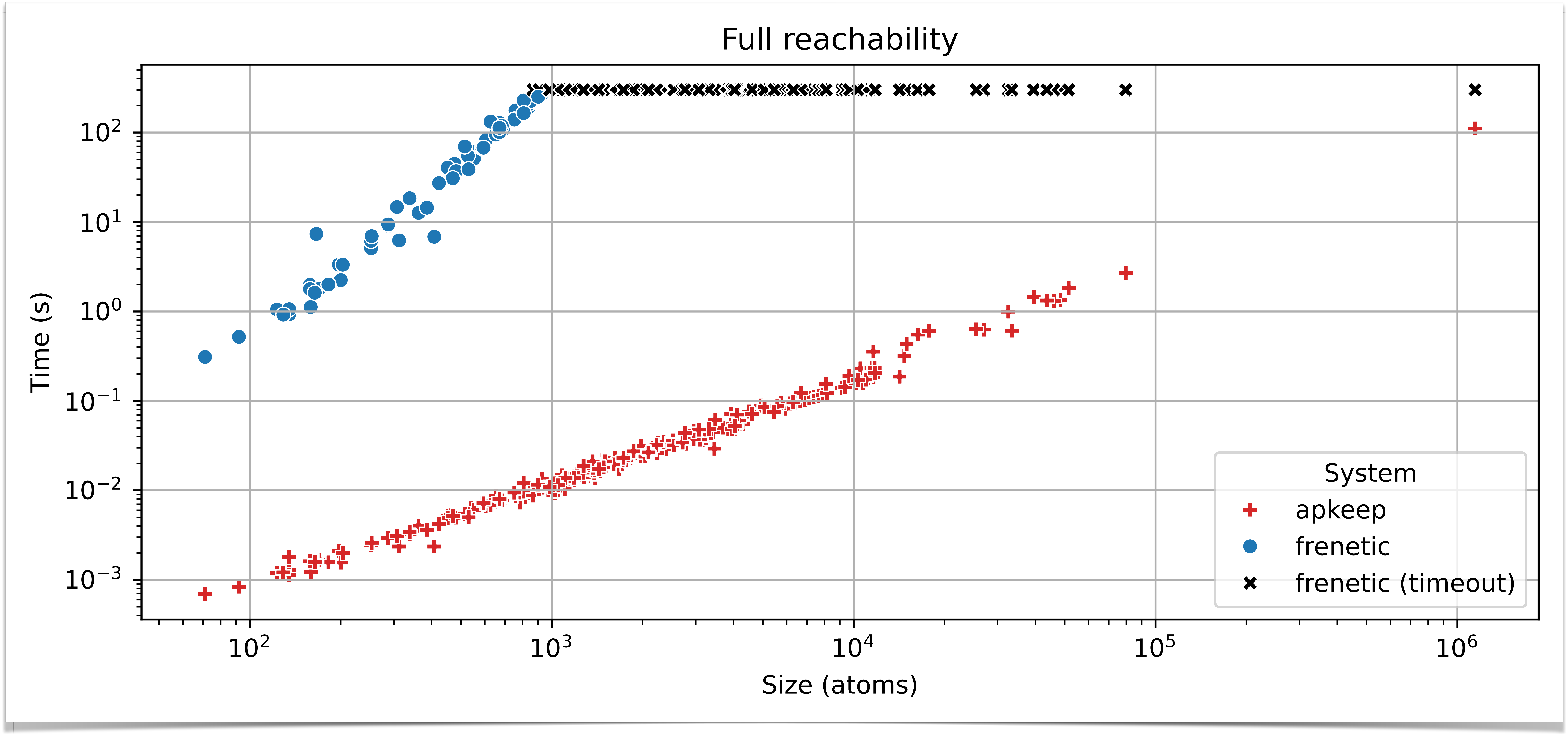
Topology

```
N0 = 0 N1 = 1 N2 = 2 N3 = 3 N4 = 4 N5 = 5
top = @pt=-1?·εU(@sw=N5?·(@pt=2?·(@sw←N4·@pt←1)U@pt=1?
·(@sw←N3·@pt←2)U@pt=0?·(@sw←N1·@pt←3))U@sw=N4?·(@pt=1?
·(@sw←N5·@pt←2)U@pt=0?·(@sw←N3·@pt←1))U@sw=N3?·(@pt=2?
·(@sw←N5·@pt←1)U@pt=1?·(@sw←N4·@pt←0)U@pt=0?
·(@sw←N1·@pt←2))U@sw=N2?·(@pt=0?·(@sw←N1·@pt←1))U@sw=N1?·(@pt=3?
·(@sw←N5·@pt←0)U@pt=2?·(@sw←N3·@pt←0)U@pt=1?·(@sw←N2·@pt←0)U@pt=0?
·(@sw←N0·@pt←0))U@sw=N0?·(@pt=0?·(@sw←N1·@pt←0)))
```

Switches

```
 @sw=N5?·(@dst=N4?·@pt←2U@dst=N3?·@pt←1U@dst=N2?·@pt←0U@dst=N1?
·@pt←0U@dst=N0?·@pt←0U@dst=N5?·@pt←-1)U@sw=N4?·(@dst=N5?
·@pt←1U@dst=N3?·@pt←0U@dst=N2?·@pt←0U@dst=N1?·@pt←0U@dst=N0?
·@pt←0U@dst=N4?·@pt←-1)U@sw=N3?·(@dst=N5?·@pt←2U@dst=N4?
·@pt←1U@dst=N2?·@pt←0U@dst=N1?·@pt←0U@dst=N0?·@pt←0U@dst=N3?
·@pt←-1)U@sw=N2?·(@dst=N5?·@pt←0U@dst=N4?·@pt←0U@dst=N3?
·@pt←0U@dst=N1?·@pt←0U@dst=N0?·@pt←0U@dst=N2?·@pt←-1)U@sw=N1?
·(@dst=N5?·@pt←3U@dst=N4?·@pt←3U@dst=N3?·@pt←2U@dst=N2?
·@pt←1U@dst=N0?·@pt←0U@dst=N1?·@pt←-1)U@sw=N0?·(@dst=N5?
·@pt←0U@dst=N4?·@pt←0U@dst=N3?·@pt←0U@dst=N2?·@pt←0U@dst=N1?
·@pt←0U@dst=N0?·@pt←-1)
```

Evaluation


Full reachability
Time (s)
Size (atoms)
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{2}
10^{3}
10^{4}
10^{5}
10^{6}
System
apkeep
frenetic
frenetic (timeout)

Symbolic automata [PLDI '24]

A *NetKAT Automaton* is a tuple M=⟨S, s_0, ε, δ⟩ where:

- …
- ε ∈ S → **Packet** → **Packet Set**
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Symbolic automata I ’24]

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There are a *lot* of packets (and even more relations on packets)!

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Idea: encode relations using a symbolic representation that is optimized for the common case–i.e., most fields are not changed

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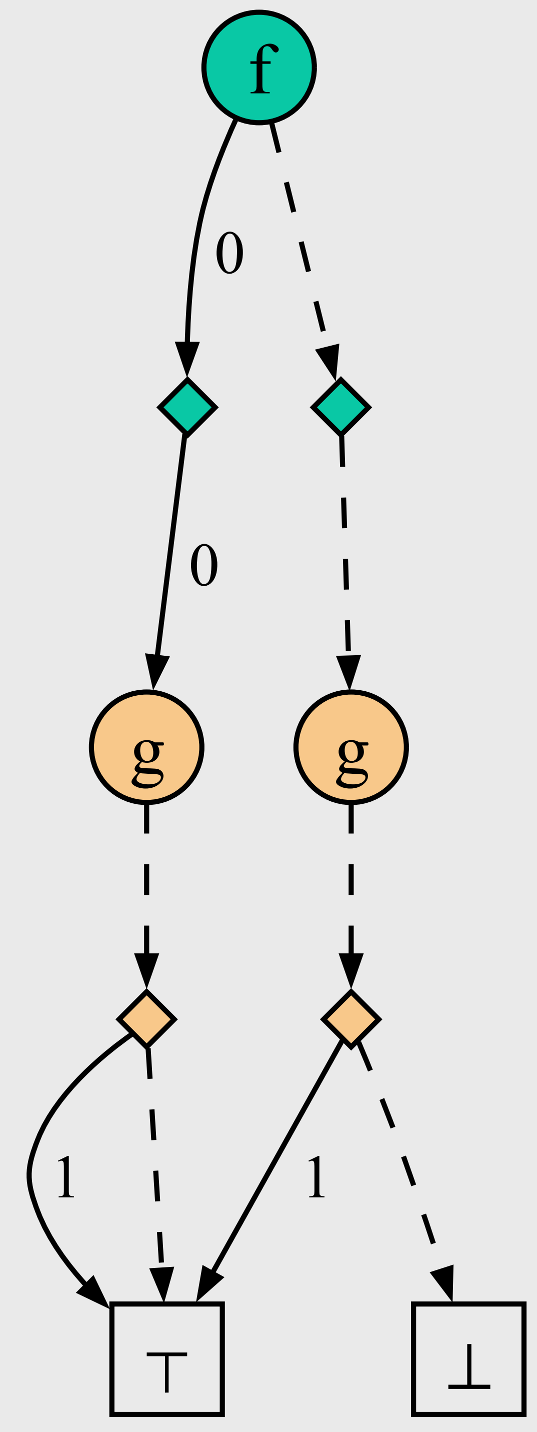
Example

```
f = 0 + g := 1
```



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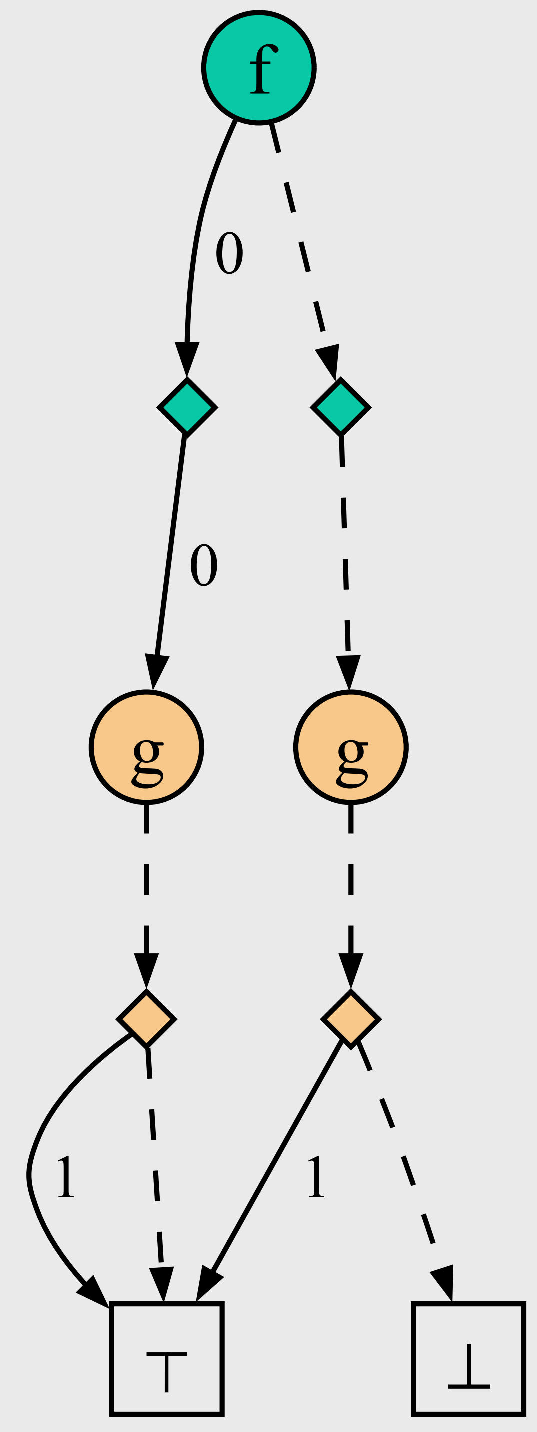
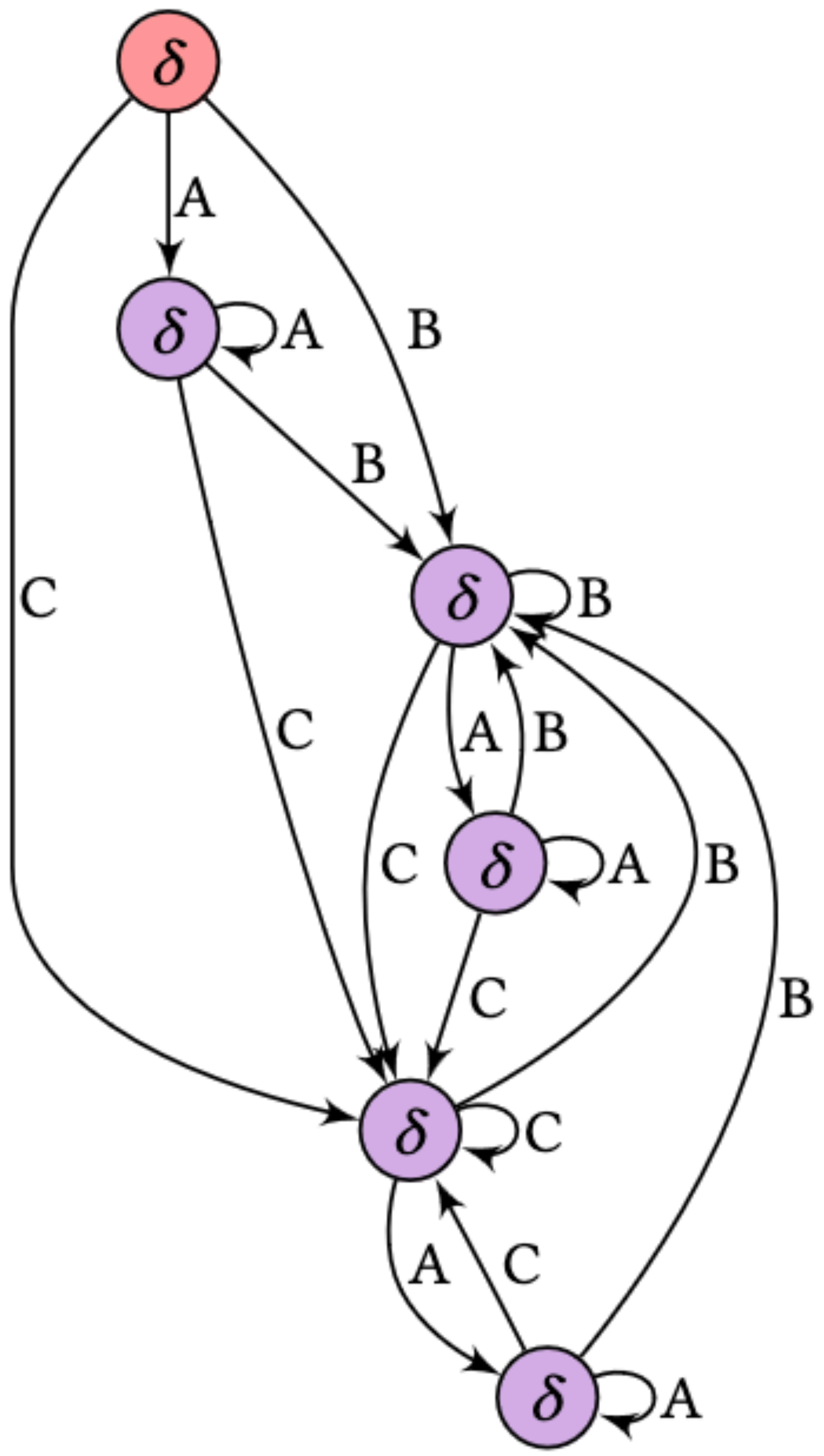
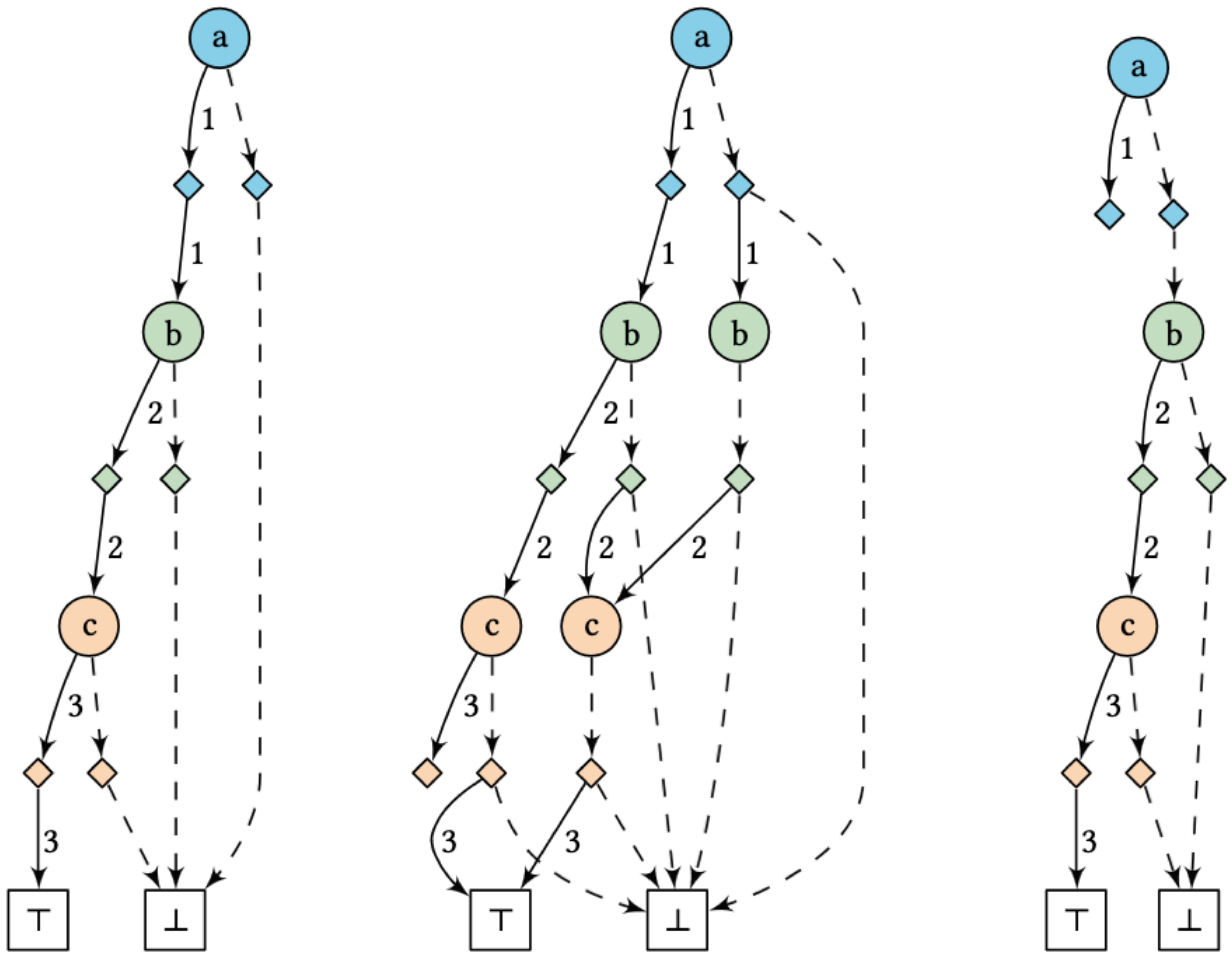


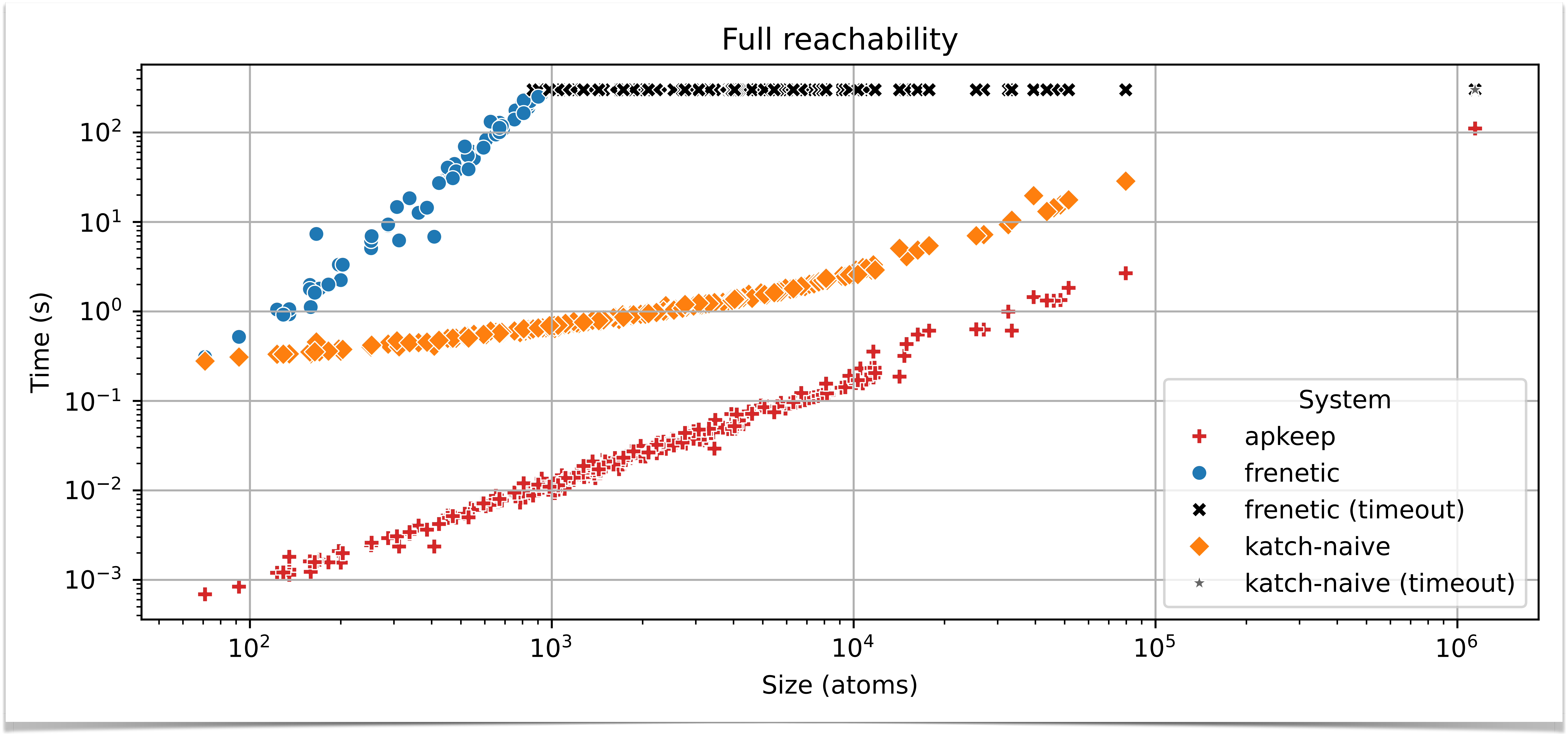
Symbolic automata



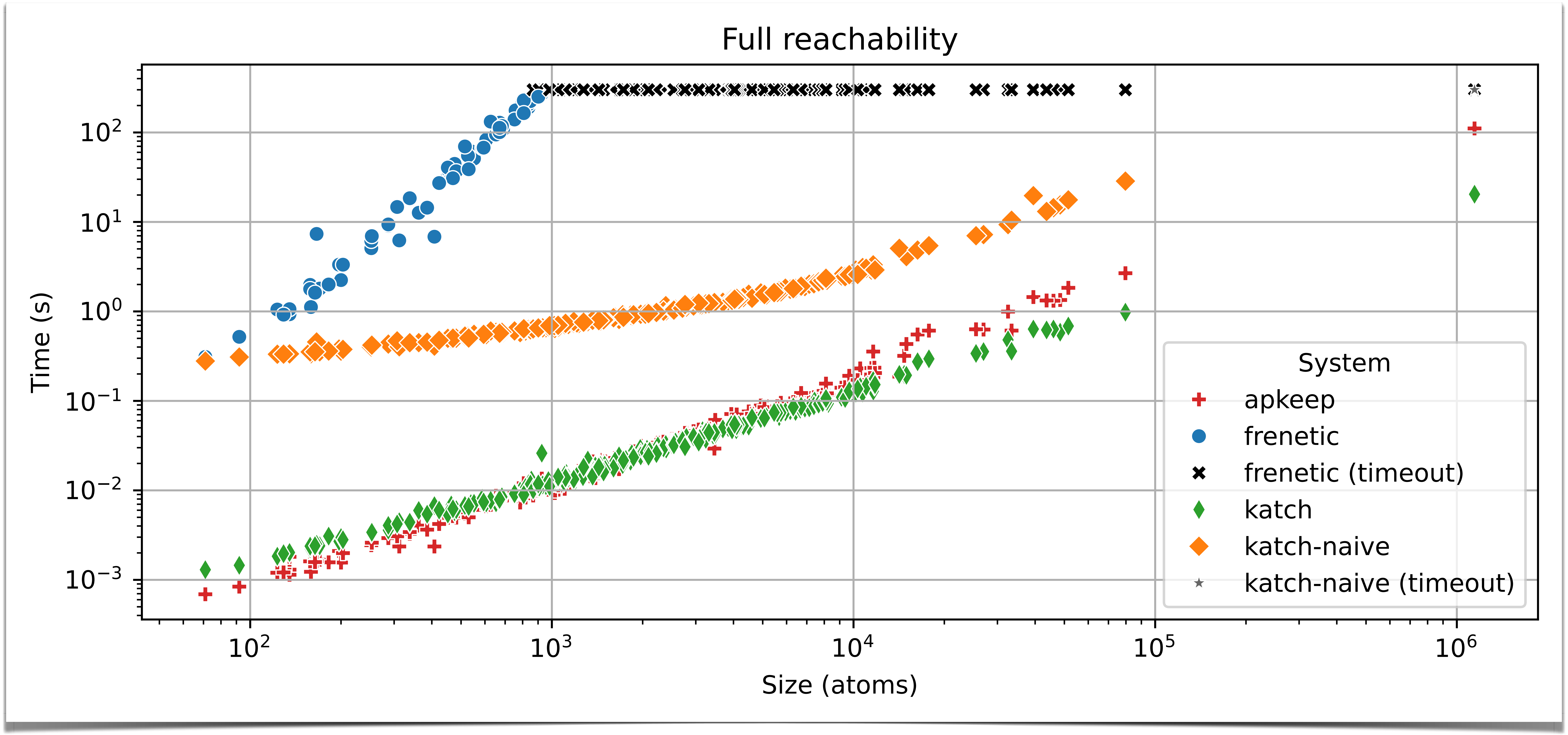


A B C

Evaluation: KATch

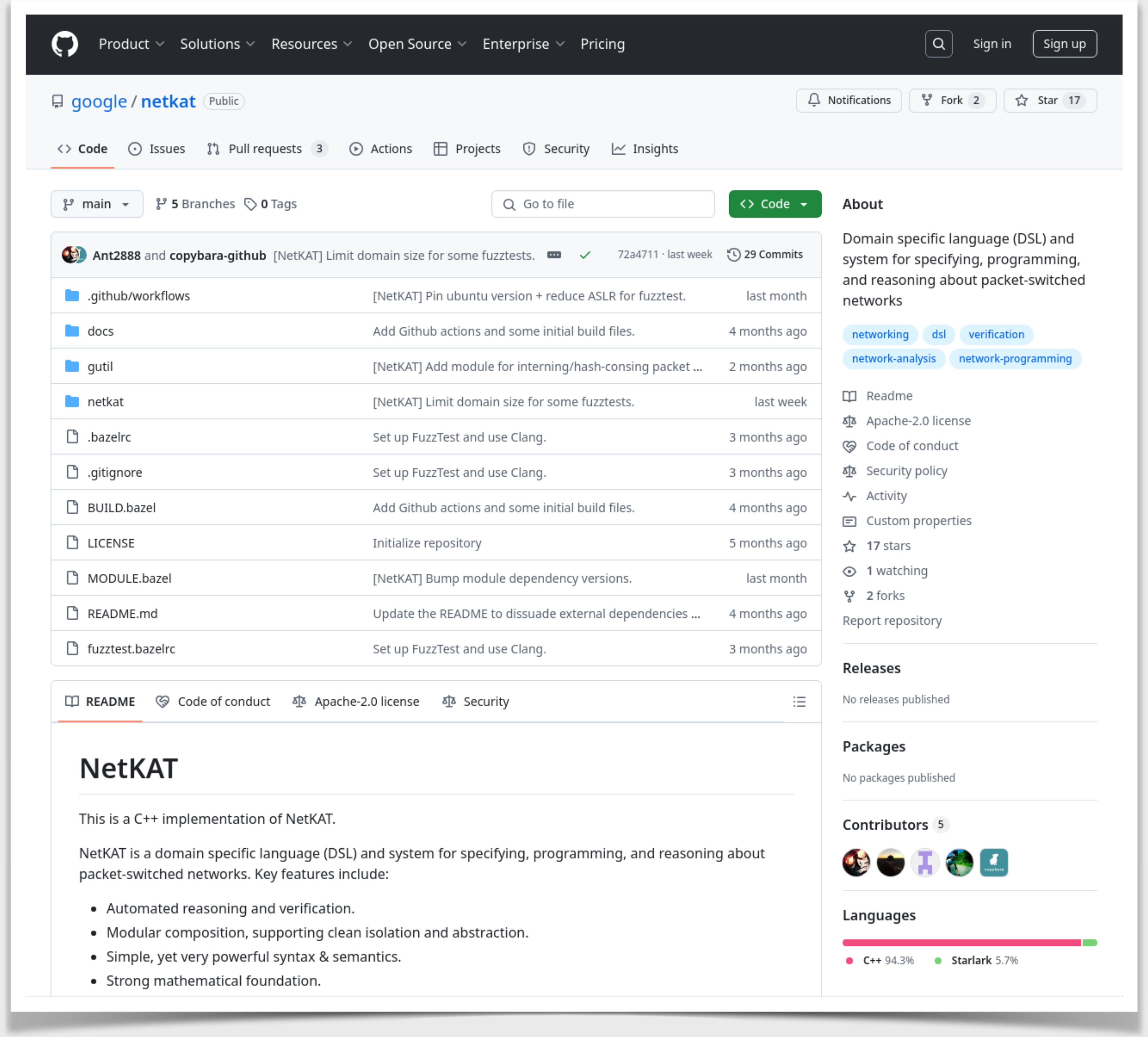


Evaluation: KATch + linear encoding



NetKAT in industry

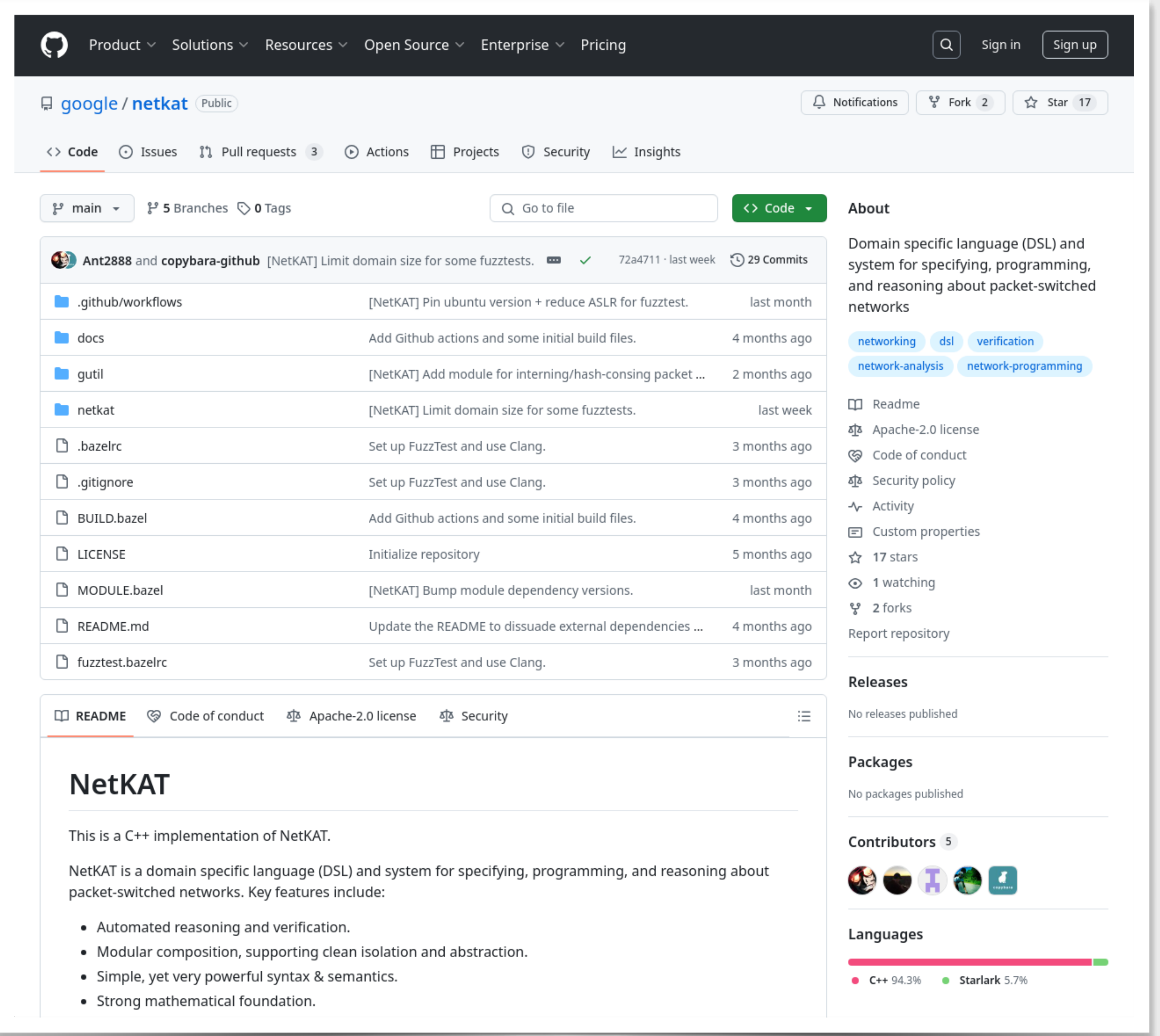
Google



C++ implementation of NetKAT for verifying cloud isolation

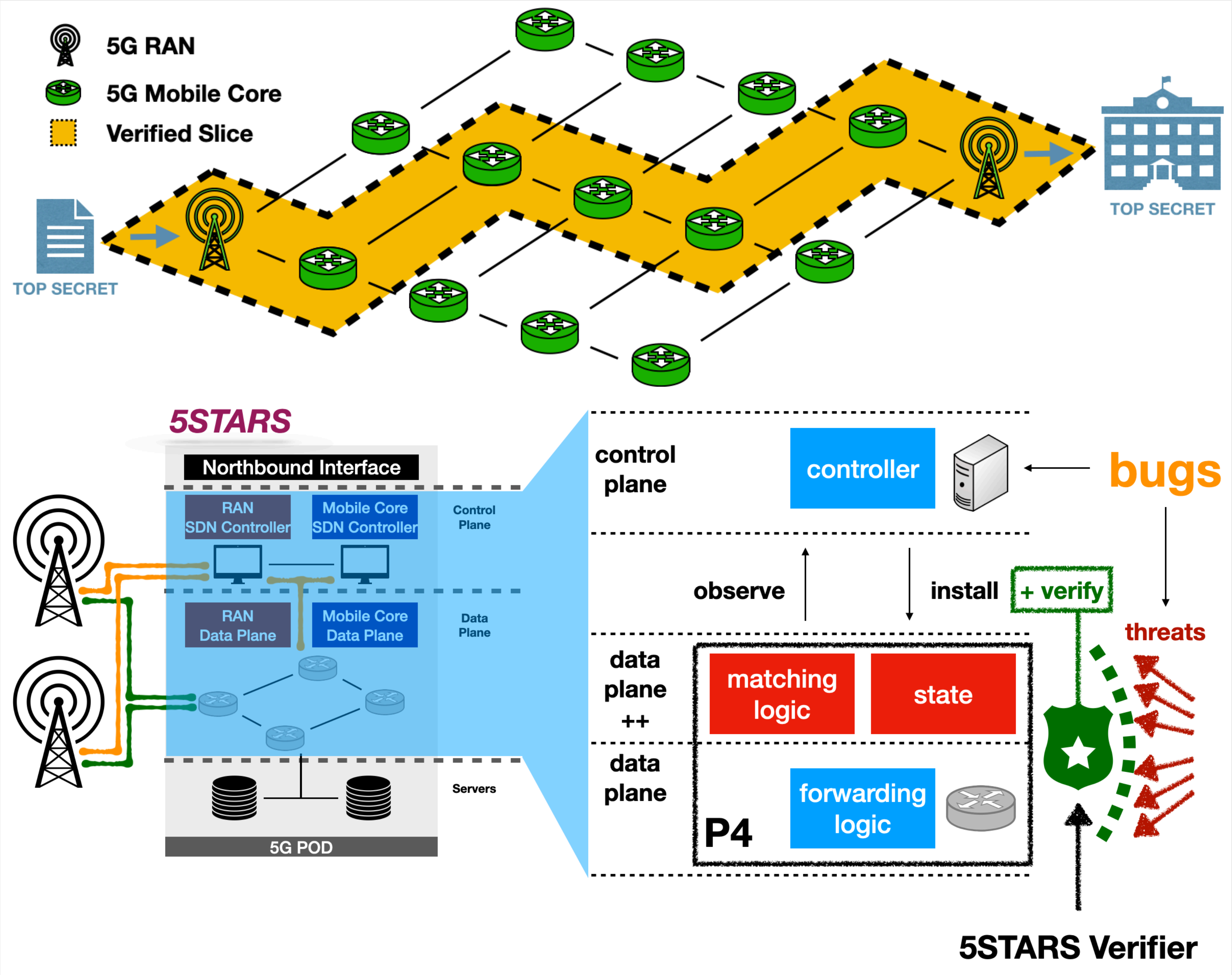
NetKAT in industry

Google



C++ implementation of NetKAT for verifying cloud isolation

galois



Haskell implementation of NetKAT for verifying secure 5G slicing

Lots of KATs

KAT with finite mutable state
LICS '14

NetKAT with temporal look around
PLDI '16

Normal Forms for Network Data Planes
CoNEXT '19

Verifying File Systems using Kleene Algebra
PLDI '19

Higher-order ProbNetKAT
POPL '20

Dynamic NetKAT
FoSSaCS '22

KA modulo theories
PLDI '22 🏆

Verifying Quantum Programs with Kleene Algebra
PLDI '22

BellNetKAT
PLDI '24

Verifying Control Flow Transformations with GKAT
POPL '25

Victory Lap

Course Goals

At the start of the semester, we set out to:

- Understand how to design languages…
- By modeling their semantics mathematically

JavaScript

```
[] + []
{} + []
[] + {}
{} + {}
```

From *Wat:*
https://www.destroyallsoftware.com/talks/wat

Looking Back

CS {4,5}110 Home Resources Schedule Syllabus Ed CMSX

Schedule

Introduction

Date	Topic	Notes	Assignments
August 25	Course Overview	slides	Introductory Survey due 8/28

Mathematical Foundations

Date	Topic	Notes	Assignments
August 27	Semantics	slides notes	
August 29	Induction	slides notes	
September 1	**Labor Day**		
September 3	Lab: Semantics	lab	
September 5	Lab: Induction	lab	A1 due 9/4

Formal Semantics

Date	Topic	Notes	Assignments
September 8	IMP *(Guest: Kozen)*	slides notes	
September 10	IMP Properties *(Guest: Kozen)*	slides notes	
September 12	Lab: IMP	lab	A2 due 9/11
September 15	Denotational Semantics	slides notes	
September 17	Program Equivalence	slides notes	
September 19	Lab: Denotational Semantics	lab	A3 due 9/18
September 22	Axiomatic Semantics	slides notes	

Program Verification

Date	Topic	Notes	Assignments
September 24	Hoare Logic	slides notes	
September 26	Lab: Axiomatic Semantics	lab handout	A4 due 9/25
September 29	Predicate Transformers	slides notes	
October 1	Separation Logic	slides notes	
October 3	Lab: Separation Logic	lab	A5 due 10/2

λ Calculus

Date	Topic	Notes	Assignments
October 6	Lambda Calculus	slides notes	
October 8	More Lambda Calculus	slides notes	
October 10	**Prelim I**		
October 13	**Fall Break**		
October 15	Definitional Translation *(Guest: Myers)*	slides notes	
October 17	Continuations *(Guest: Myers)*	slides notes	
October 20	Fixed-point Combinators	slides notes	
October 22	de Bruijn Notation and Combinators	slides notes	
October 24	Lab: Lambda-Calculus	lab	A6 due 10/23

Type Systems

Date	Topic	Notes	Assignments
October 27	Type Systems	slides notes	
October 29	Advanced Types	slides notes	
October 31	Lab: Type Systems	lab	A7 due 10/30
November 3	Polymorphism	slides notes	
November 5	Making OCaml Safe for Performance Engineering *(Guest: Minsky)*		
November 7	Lab: Polymorphism	lab	A8 due 11/8

Type Theory

Date	Topic	Notes	Assignments
November 10	**Prelim II**		
November 12	Dependent Types and Type Theory *(Guest: Barbone)*	slides	
November 14	Lab:Type Theory *(Guest: Barbone)*	lab	
November 17	Normalization and Logical Relations	slides notes	
November 19	**Foster out of town**		
November 21	Lab:Logical Relations	lab	A9 due 11/20

Advanced Topics

Date	Topic	Notes	Assignments
November 24	Logic Programming	slides notes code	
November 26	**Thanksgiving Break**		
November 28	**Thanksgiving Break**		
December 1	Lenses	slides	
December 3	Program Synthesis	slides	
December 5	Lab: Domain-Specific Languages		

Mathematical Foundations

Main Topics

- Sets
- Relations
- Functions
- Inductive Proof

Induction Principle

Every inductive set has an analogous principle.

To prove $\forall a.\ P(a)$ we must establish several cases.

- Base cases: $P(a)$ holds for each axiom

$$\frac{}{a \in A}$$

- Inductive cases: For each non-axiom inference rule

$$\frac{a_1 \in A \quad \ldots \quad a_n \in A}{a \in A}$$

if $P(a_1)$ and … and $P(a_n)$ then $P(a)$.

Formal Semantics

Main Topics

- Operational
- Denotational
- Axiomatic
- Fixed points

Denotational Semantics for IMP Commands

$$\mathcal{C}[\![\textbf{skip}]\!] = \{(\sigma,\sigma)\}$$

$$\mathcal{C}[\![x := a]\!] = \{(\sigma, \sigma[x \mapsto n]) \mid (\sigma, n) \in \mathcal{A}[\![a]\!]\}$$

$$\mathcal{C}[\![c_1; c_2]\!] = \\ \{(\sigma,\sigma') \mid \exists\sigma''.\ ((\sigma,\sigma'') \in \mathcal{C}[\![c_1]\!] \wedge (\sigma'',\sigma') \in \mathcal{C}[\![c_2]\!])\}$$

$$\mathcal{C}[\![\textbf{if } b \textbf{ then } c_1 \textbf{ else } c_2]\!] = \\ \{(\sigma,\sigma') \mid (\sigma, \textbf{true}) \in \mathcal{B}[\![b]\!] \wedge (\sigma,\sigma') \in \mathcal{C}[\![c_1]\!]\} \ \cup \\ \{(\sigma,\sigma') \mid (\sigma, \textbf{false}) \in \mathcal{B}[\![b]\!] \wedge (\sigma,\sigma') \in \mathcal{C}[\![c_2]\!]\}$$

$$\mathcal{C}[\![\textbf{while } b \textbf{ do } c]\!] = \mathit{fix}(f) \\ \text{where } F(f) = \{(\sigma,\sigma) \mid (\sigma, \textbf{false}) \in \mathcal{B}[\![b]\!]\} \ \cup \\ \{(\sigma,\sigma') \mid (\sigma, \textbf{true}) \in \mathcal{B}[\![b]\!] \wedge \exists\sigma''.\ ((\sigma,\sigma'') \in \mathcal{C}[\![c]\!] \wedge \\ (\sigma'',\sigma') \in f\}$$

Program Verification

Main Topics

- Partial vs. Total Correctness
- Hoare Logic
- Verification Conditions

Weakest Preconditions

$$
\begin{aligned}
wlp(\textbf{skip}, P) &= P \\
wlp(x := a, P) &= P[a/x] \\
wlp((c_1; c_2), P) &= wlp(c_1, wlp(c_2, P)) \\
wlp(\textbf{if } b \textbf{ then } c_1 \textbf{ else } c_2, P) &= (b \implies wlp(c_1, P)) \land \\
&\quad\; (\neg b \implies wlp(c_2, P)) \\
wlp(\textbf{while } b \textbf{ do } c, P) &= \textstyle\bigwedge_i F_i(P)
\end{aligned}
$$

where

$$
\begin{aligned}
F_0(P) &= \textbf{true} \\
F_{i+1}(P) &= (\neg b \implies P) \land (b \implies wlp(c, F_i(P)))
\end{aligned}
$$

λ-Calculus

Main Topics

- Reduction Strategies
- Encodings
- Fixed Points
- Definitional Translation

Laziness

Consider the call-by-name λ-calculus...

Syntax

$$\begin{aligned} e &::= x \\ &\mid e_1\ e_2 \\ &\mid \lambda x.\ e \\ v &::= \lambda x.\ e \end{aligned}$$

Semantics

$$\frac{e_1 \to e_1'}{e_1\ e_2 \to e_1'\ e_2} \qquad \frac{}{(\lambda x.\ e_1)\ e_2 \to e_1\{e_2/x\}}\ \beta$$

Type Systems

Main Topics

- Typing Relations
- Progress
- Preservation
- Polymorphism

Simply-Typed Lambda Calculus

Static Semantics

$$\frac{}{\Gamma \vdash n : \mathbf{int}}\ \textsc{T-Int} \qquad \frac{}{\Gamma \vdash () : \mathbf{unit}}\ \textsc{T-Unit}$$

$$\frac{\Gamma \vdash e_1 : \mathbf{int} \quad \Gamma \vdash e_2 : \mathbf{int}}{\Gamma \vdash e_1 + e_2 : \mathbf{int}}\ \textsc{T-Add}$$

$$\frac{\Gamma(x) = \tau}{\Gamma \vdash x : \tau}\ \textsc{T-Var} \qquad \frac{\Gamma, x{:}\tau \vdash e : \tau'}{\Gamma \vdash \lambda x{:}\tau.\, e : \tau \to \tau'}\ \textsc{T-Abs}$$

$$\frac{\Gamma \vdash e_1 : \tau \to \tau' \quad \Gamma \vdash e_2 : \tau}{\Gamma \vdash e_1\, e_2 : \tau'}\ \textsc{T-App}$$

Type Theory

Main Topics

- Dependent Types
- Normalization
- Logical Relation

Logical Relation

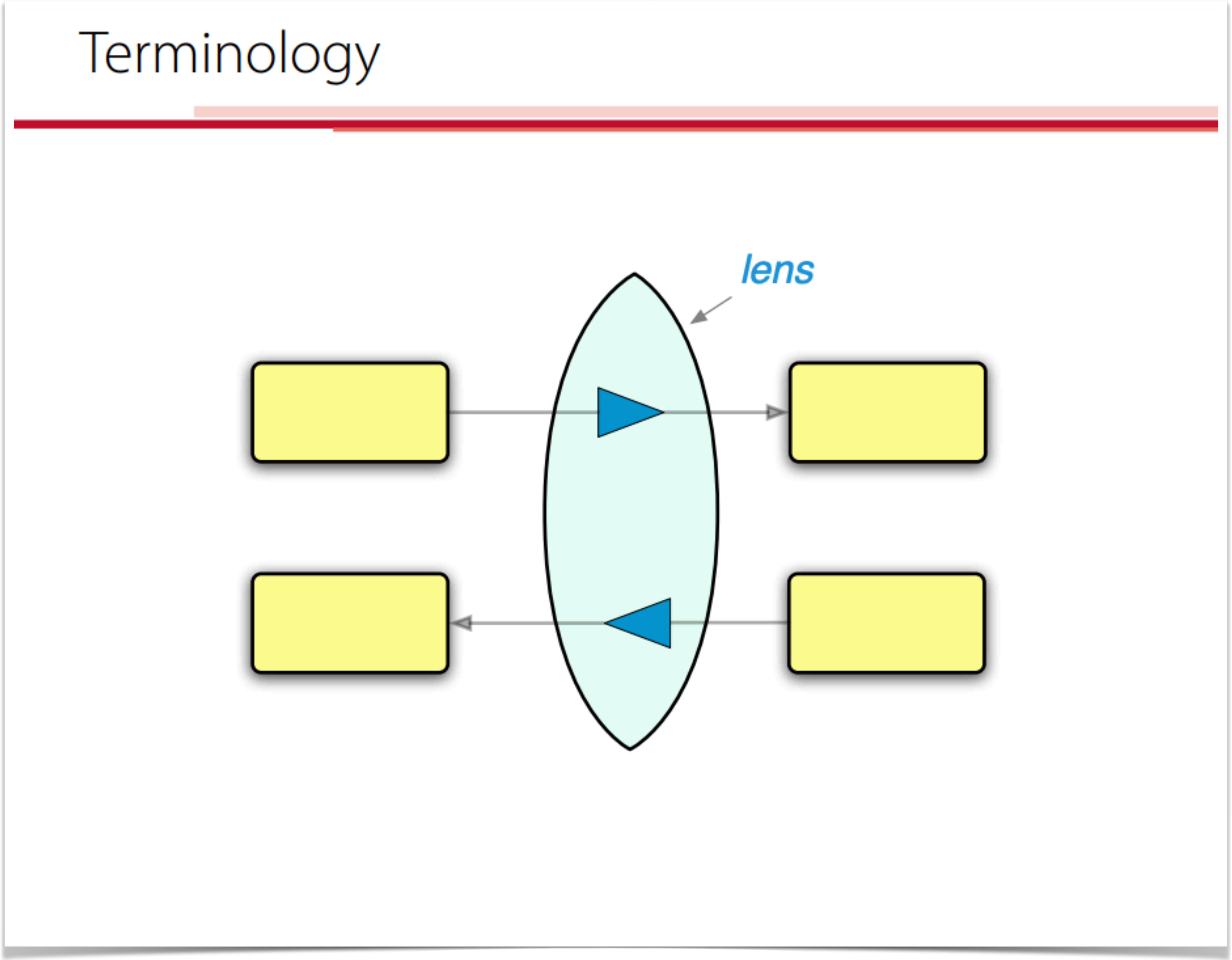
Definition (Logical Relation)

- $R_{\mathbf{unit}}(e)$ iff $\vdash e : \mathbf{unit}$ and e halts.
- $R_{\tau_1 \to \tau_2}(e)$ iff $\vdash e : \tau_1 \to \tau_2$ and e halts, and for every e' such that $R_{\tau_1}(e')$ we have $R_{\tau_2}(e\, e')$.

Advanced Topics

Main Topics

- DSLs
- Logic Programming
- Program Synthesis



Next Steps

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Courses: CS 4120 (Compilers), CS 6110 (Advanced PL), CS 6120 (Advanced Compilers), CS 6117 (Category Theory)

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Research: BURE, ACSU Research Night, CS 4999

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Research: BURE, ACSU Research Night, CS 4999

After Cornell: Compilers, Formal Verification, Grad School

Thank You!


Ithaca
Lausanne
EPFL