CS 4110
Programming Languages & Logics

Lecture 36
Program Synthesis

The Dream

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- A few years ago, this would have sounded like science fiction.
- But with LLMs (ChatGPT, Cursor, etc.) it has become totally common!
- How do we know a synthesized program is correct?

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SYNTHESIS: DREAMS => PROGRAMS

by

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Program Synthesis

Goal: Generate a program satisfying constraints

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- Logical specifications
- Partial programs (“sketches”)
- Input–output examples

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- Inductive synthesis

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Techniques

- Deductive synthesis
- Inductive synthesis
  - Enumeration (top-down/bottom-up)
  - CEGIS
  - Sketching

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Key Challenge: search space is *huge*.

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Common Approaches

- Limit space of programs
- Harness symbolic representations
- Use SAT/SMT solvers
- Rely on domain heuristics

Example: Synthesizing Regular Expressions

Definition (Problem Statement)

Given positive (P) and negative (N) examples, find R s.t. $P \subseteq [\![R]\!]$ and $N \cap [\![R]\!] = \emptyset$.

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Examples

```
01,+
X01,+
XX01,+
X0,-
X11,-
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$\Longrightarrow$

Solution

```
(0 + 1)* . 0 . 1
```

Defining the Search Space

Definition (Partial Regular Expressions)

R	$::=$	$\square$	*Hole*
	$\mid$	$\emptyset$	*Empty Set*
	$\mid$	ϵ	*Empty String*
	$\mid$	c	*Character*
	$\mid$	$R_1 + R_2$	*Union*
	$\mid$	$R_1 \cdot R_2$	*Concatenation*
	$\mid$	R^*	*Kleene Star*

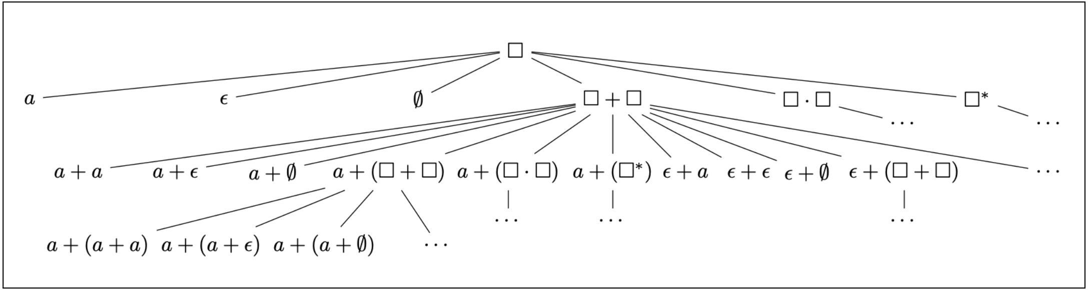
Defining the Search Space

Definition (Expansion Rules)

$$\begin{aligned}
\square &\rightarrow \emptyset \\
\square &\rightarrow \epsilon \\
\square &\rightarrow c \\
\square &\rightarrow \square + \square \\
\square &\rightarrow \square \cdot \square \\
\square &\rightarrow \square *
\end{aligned}$$

Plus the “obvious” congruence rules for $+$, $\cdot$, and *.

Visualizing the search space



https://cs.stanford.edu/~minalee/pdf/gpce2016-alpharegex.pdf

Synthesis Algorithm

Definition (Naive Synthesis)

```
synthesize(P, N):
  W := { □ }
  repeat
    pick R from W
    if solution(R, P, N) then
      return R
    else
      W := W ∪ { R' | R → R' }
  until W = {}
```

Heuristic Optimizations

Prioritize small states

- Try smaller programs before bigger ones.

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Avoid redundancy

- Skip states irrelevant for covering positive examples.

Cost function

Prioritize small states: Try smaller programs before bigger ones.

Definition (Cost)

$$
\begin{aligned}
\mathcal{C}(\square) &\triangleq c_1 \\
\mathcal{C}(\emptyset) &\triangleq c_1 \\
\mathcal{C}(\epsilon) &\triangleq c_1 \\
\mathcal{C}(c) &\triangleq c_1 \\
\mathcal{C}(R_1 + R_2) &\triangleq \mathcal{C}(R_1) + \mathcal{C}(R_2) + c_2 \\
\mathcal{C}(R_1 \cdot R_2) &\triangleq \mathcal{C}(R_1) + \mathcal{C}(R_2) + c_3 \\
\mathcal{C}(R*) &\triangleq \mathcal{C}(R) + c_4
\end{aligned}
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\end{aligned}
$$

Typically, pick $c_2 > c_3$ so that concatenation is preferred over union.

Equivalence

Recognize equivalent states: Normalize regexes using rewrite rules

Definition (Regex Equivalences)

$$\emptyset \cdot R \equiv \emptyset \equiv R \cdot \emptyset$$
$$\epsilon \cdot R \equiv R \equiv R \cdot \epsilon$$
$$(R_1 \cdot R_2) \cdot R_3 \equiv R_1 \cdot (R_2 \cdot R_3)$$
$$(R_1 + R_2) + R_3 \equiv R_1 + (R_2 + R_3)$$
$$R_1 + R_2 \equiv R_2 + R_1$$
$$R_1 \cdot R_2 + R_1 \cdot R_3 \equiv R_1 \cdot (R_2 + R_3)$$
$$R_1 \cdot R_2 + R_3 \cdot R_2 \equiv (R_1 + R_3) \cdot R_2$$
$$R \cdot R* \equiv R*$$
$$\emptyset* \equiv \epsilon \equiv \epsilon*$$

Pruning

Prune dead states: Fill holes and over/under approximate

Definition (Over/Under Approximations)

$$\widehat{\square} \triangleq (c_1 + \ldots c_n)*$$
$$\widetilde{\square} \triangleq \emptyset$$

(And homomorphically for other constructs...)

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Definition (Dead State)

$$dead(R) \triangleq \exists p \in P.\ p \notin [\![\widehat{R}]\!] \vee \exists n \in N.\ n \in [\![\widetilde{R}]\!]$$

Redundancy

Avoid redundancy: Skip states irrelevant for covering positive examples.

Example

Suppose $0 \notin P$. Then $0 + \square$ is redundant as $\square$ suffices.

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Suppose $P \triangleq \{0, 01, 011, 0111\}$. Then $0 * \cdot \square$ is redundant as $\square$ suffices.

The details for recognizing redundant states are slightly involved; see the original paper on AlphaRegex (GPCE '16) for full details.

Demo

Properties

Definition (Soundness)

If $R \in \texttt{synthesize(N,P)}$ then $P \subseteq [\![R]\!]$ and $N \cap [\![R]\!] = \emptyset$.

Definition (Completeness)

If $\texttt{synthesize}(N, P)$ fails then $\nexists R$ such that $P \subseteq [\![R]\!]$ and $N \cap [\![R]\!] = \emptyset$.

Note that synthesis can never fail, so completeness holds trivially...

Counterexample-Guided Inductive Synthesis

Definition (CEGIS)

```
cegis(spec):
  cexs := ∅
  while true:
    candidate := generate(spec, cexs)
    if verify(spec, candidate):
      return candidate
    else:
      obtain counterexample e
      cexs := cexs ∪ { e }
```

More Approaches

Bottom-up Enumeration

- Generate candidate programs from small to large
- Similar to dynamic programming

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- E-graphs

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Stochastic Search

- Based on probabilistic algorithms
- Markov Chain Monte Carlo

Even More Approaches

Type-Guided Synthesis

- Prune ill-typed programs
- Use refinement types to capture semantic constraints

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Component Discovery

- Meta-learning to generate a DSL
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LLM-Based Synthesis

- Rejection sampling
- Constrained decoding