

①

$$\{x = m \wedge y = n \wedge 0 \leq n\}$$

$\{I\}$   
While  $0 < y$

$$\{I \wedge 0 < y\} \Rightarrow$$

$$x := x + 1;$$

$$\{P_1\}$$

$$y := y - 1$$

$$\{I \wedge 0 < y\} \Rightarrow$$

$$\{x = m + n\}$$

$$x + y = m + n$$

$$x = m + n - y$$

$$I ::= (x = m + n - y) \wedge 0 \leq y$$

$$P_1 ::= I[y-1 / y]$$

$$= (x = m + n - (y-1)) \wedge (0 \leq y-1)$$

$$P_2 ::= P_1[x+1 / x]$$

$$= (x+1 = m + n - y + 1) \wedge (0 \leq y-1)$$

$$P = ~~x \neq 0~~ \text{ true}$$

$$Q = x = 0$$

$$\{ \text{true} \}$$

$$\{ \text{true} \}$$

While  $x \neq 0$  do

$$\{ \text{true} \wedge x \neq 0 \} \Rightarrow \dots$$

$$x := x - x$$

$$\{ \text{true} \}$$

$$\{ \text{true} \wedge x = 0 \} \Rightarrow$$

$$\{ x = 0 \}$$

$$\{ \text{true} \}$$

$$x \geq 0$$

2

$\{x = n \wedge 0 \leq n\} \Rightarrow$   
 $\{I \ [1/y]\}$

$y := 1$

$\{I\}$   
while  $(0 < x)$  do

$\{I \wedge 0 < x\} \Rightarrow$   
 $\{I \ [y * 2] \ [x - y * x]\}$

$x := x - y * x$   
 $\{I \ [y * 2] \ [y]\}$

$y := y * 2$

$\{I \wedge 0 \leq x\} \Rightarrow$

$\{y = 2^n\}$

$I ::= y = 2^n / 2^x \wedge 0 \leq x \wedge y \geq 1$   
 $= y = 2^{n-x} \wedge 0 \leq x \wedge y \geq 1$