Hand in your assignment next Wednesday in class. Type your homework or write very neatly!

1. Find an appropriate loop invariant for the program below and use it to prove that the following partial correctness assertion statement holds:

$$\{x = 1 \land y = i\} \ \ \mathsf{while}\ (y \neq 0)\ \mathsf{do}\ (y := y - 1;\ x := 2 * x) \ \ \{x = 2^i\}$$

2. The following program is aimed at approximating the square root of a number n. The program uses two variables x and y to describe an interval $[x, y]$ that holds the desired square root value. The program starts with interval $[0, n + 1]$; at each iteration it halves the interval, decides whether the square root lies in the lower half $[x, (x + y)/2]$ or in the upper half $[(x + y)/2, y]$, and adjusts the interval bounds accordingly. The execution finishes when the interval becomes small enough (i.e., has length 1). Variable x holds the final result.

```
x := 0;
y := n + 1;
while (x + 1 ≠ y) do (
    z := (x + y)/2;
    if (z * z > n) then y := z else x := z
)
```

If we refer to the program above as *SquareRoot*, use Hoare rules to prove the correctness of the following triple:

$$\{n \geq 0\} \ \ \mathit{SquareRoot} \ \ \{x^2 \leq n < (x + 1)^2\}$$

3. Suppose we tweak the Hoare rule for while loops as follows:

$$\frac{\{P \land b\}\ c\ \{P\}}{\{P\}\ \mathsf{while}\ b\ \mathsf{do}\ c\ \{P\}}$$

- Is the new rule sound? Explain.
- Are there valid partial correctness triples that cannot be derived using this rule and the other Hoare rules? Explain (and show an example triple, if any).