Hand in your assignment next Friday in class. Type in your homework, or write *very neatly.*

1. Prove that, if expressions e, e', state s, and integer n are such that $\langle e, s\rangle \rightarrow \langle e', s\rangle$ and $\langle e', s\rangle \Downarrow n$, then $\langle e, s\rangle \Downarrow n$.

   Note: make sure you clearly indicate the property you are trying to prove and what kind of induction you're using.

2. Redefine the small step evaluation relation for expressions presented in class to force the evaluation from right to left (instead of left to right). Indicate which rules need to be changed and show the new rules.

3. Suppose we replace the following rule in the evaluation of expressions:

$$\frac{\langle e_2, s\rangle \rightarrow \langle e_2', s\rangle}{\langle n + e_2, s\rangle \rightarrow \langle n + e_2', s\rangle} \text{ (rplus)}$$

   with a new axiom:

$$\frac{}{\langle e_1 + e_2, s\rangle \rightarrow \langle e_2 + e_1, s\rangle} \text{ (new)}$$

   (a) Does the progress property proved in class still hold? If yes, show the changes in the proof. If no, show a counterexample.

   (b) With the new rule, there are evaluations that go on forever. Give an example of such an evaluation.

   (c) Add side conditions to the new axiom to ensure that there cannot be infinite evaluations.

4. We want to augment the expression semantics presented in class, such that the evaluation of an expression yields not only the numeric value of the expression, but also the cost of evaluating the expression. The cost model is defined as follows: each addition has a cost of 1, each multiplication a cost of 2, and each variable access a cost of 3. For instance, expression $(y + 2) * (y + 1)$ has a total cost of 10. Define an appropriate large-step semantics for the evaluation of expressions with costs.