CS 410 Summer 2000 **Quiz 8** **Thursday July 20**

Name: ______________________________ ID: ______________

You may also get some points back on your prelim grade with this quiz.

1. Recall Stooge-Sort from homework assignment 3:

```
Stooge-Sort (A,i,j)
 1  if A[i] > A[j]
 2     swap A[i] and A[j]
 3  if i+1 >= j
 4     return
 5  k = floor((j-i+1)/3)        // round down
 6  Stooge-Sort (A,i,j-k)       // First two-thirds
 7  Stooge-Sort (A,i+k,j)       // Last two-thirds
 8  Stooge-Sort (A,i,j-k)       // First two-thirds again
```

Complete the following proof of the correctness of Stooge-Sort.

Proof is by induction on the size of the array A.

(base case: 1 and 2 element arrays) When the length of input is 1, Stooge-Sort does nothing before returning, which is correct since an array of size 1 is already sorted. When the length of input is 2, it puts these 2 elements into the correct order, then returns.

Induction Hypothesis: Stooge-Sort sorts correctly for input of any size between 1 and $n-1$.

(induction step) Consider input of size $n : A = (a_1, a_2, ...a_n)$*, where* $n \geq 3$.
By the induction hypothesis, lines 6, 7, and 8 each correctly sort two-thirds of A. We need to show that this combines to correctly sort the entire array.

One detail generally missed is to note that each two-thirds which is sorted is in fact $\lceil 2n/3 \rceil$ elements. The fact that it is the ceiling and not the floor is significant to there being enough overlap, but we will ignore that in the remainder of this proof.

Let $B = (a_1, a_2, ..., a_{2n/3})$.

After line 6 (CLM-sort(A,1,2n/3)), the first one-half elements of B (the first one-third elements of A) can never belong in the last one-third elements in the sorted order of A, since we know that the last one-half elements of B are all larger than these.

Therefore the second call to CLM-sort on line 7 involving the last one-half elements of sorted array B and the last one-third of A fixes the last one-third part of A correctly.

Finally, the third call to CLM-sort on line 8 fixes the first two-thirds.

Hence CLM-sort correctly sorts the input data.