CS 410 Summer 2000 **Quiz 2** **Friday June 30**

Name: ______________________________ ID: ________________

Theorem 4.1 (Master theorem)
Let $a \geq 1$ and $b > 1$ be constants, let $f(n)$ be a function, and let $T(n)$ be defined on the nonnegative integers by the recurrence

$$T(n) = a\,T(n/b) + f(n),$$

where we interpret n/b to mean either $\lfloor n/b \rfloor$ or $\lceil n/b \rceil$.
Then $T(n)$ can be bounded asymptotically as follows.

1. If $f(n) = O(n^{\log_b a - \epsilon})$ for some constant $\epsilon > 0$, then $T(n) = \Theta(n^{\log_b a})$.
2. If $f(n) = \Theta(n^{\log_b a})$, then $T(n) = \Theta(n^{\log_b a} \lg n)$.
3. If $f(n) = \Omega(n^{\log_b a + \epsilon})$ for some constant $\epsilon > 0$,
   and if $af(n/b) \leq cf(n)$ for some constant $c < 1$ and all sufficiently large n,
   then $T(n) = \Theta(f(n))$.

For each question, indicate which case of the master theorem applies:

1. Case 1 applies
2. Case 2 applies
3. Case 3 applies

0. None of them (the master theorem does not apply)

(a) $T(n) = 9\,T(n/3) + n^2$ Answer: 2

Justification: $f(n) = n^2$ here, and $n^2 = \Theta(n^{\log_b a}) = n^2$.

(b) $T(n) = 3\,T(n/3) + n\,\lg n$ Answer: 0

Justification: $n \lg n = \Omega(n)$. but not $\Omega(n^{1+\epsilon})$ for any $\epsilon > 0$, since $\lg n = o(n^k)$ for all $k > 0$ as described in the text and discussed in lecture.

(c) $T(n) = 5\,T(n/4) + n\,\lg n$ Answer: 1

Justification: $\lg n = o(n^k)$ for all $k > 0$, so $n\,\lg n = o(n\,n^k)$

(d) $T(n) = T(n/2) + 2\,T(n/4) + n$ Answer: 0

Justification: This is not in the correct format, so the theorem doesn't apply.