

The *Corrected* Wood Shop

CS 410 Summer 2000 Homework Assignment 2 Due 11:30 AM, Tuesday July 11

Important note: This is only a portion of homework 2, not the complete assignment.

Wilson has set up shop in his garage. Literally. He has turned his hobby of carpentry into an old-fashioned business he calls Wood Row, which he runs out of his garage. It was the logical place for it. It's not so easy to move a table saw, or much of the other equipment he needs.

There are ten items he produces, which range from simple shelves to elaborate desks.¹ Ever since he advertised with a "Save the Rain Forest" web site, he's been overloaded with job orders (go figure). He hired someone to help him, but since he has only one table saw, he's limited as to how much he can produce. He wrote out the dimensions of each piece of wood needed for each item for Emily (his assistant) to use, each item described on a separate page.

He cut back his advertising to once a week, since in a week he can usually complete all the orders he gets from a day's advertising. Emily used to go through her book of directions from beginning to end, cutting enough wood for the quantity of each item ordered (i.e., if 8 beds were ordered, she would cut wood for those before moving on to cutting wood for the boxes ordered). She had arranged her instruction book alphabetically, so soon they noticed that the bed customers were quite happy with the turn-around time, but the trunk customers were threatening to bring their business elsewhere if they couldn't get their trunks sooner, and the shelf customers just couldn't understand how a simple shelf could take so long to produce.

So, Emily began researching scheduling theory. She came in after a weekend at the library, and told Wilson that if he would assign a due date to each job, then she would optimize the order in which she cut the wood. If it were possible to have all jobs completed by their due date, she would do that. If such a schedule were not possible, then she would essentially add a little to each due date until it was possible to complete them all by their modified due date. This had the effect of minimizing how late the latest job would be (here *lateness* is measured by how much time after its due date a job is completed). In scheduling theory, this criteria is called **minimum maximum lateness**, and an optimal algorithm for this criteria is called the **earliest due date rule (EDD)**, which simply schedules jobs in order of their due dates (earliest due date first, etc.).

Customer relations improved dramatically. Still, Wilson wondered about this algorithm. Since shelves could be done so quickly, he thought they ought to be able to produce some of those early, thus quickly making some customers happy without significantly impacting other customers. Other problems surfaced with how due dates were assigned, and after a month he asked Emily if there weren't another algorithm they could try.

The next Monday Emily returned with a dramatically new plan. Instead of due dates, they would assign priorities (weights) to each job. The new criterion would be to minimize the

¹The full list of items include beds, boxes, chairs, chests, desks, doors, shelves, tables, toys, and trunks.

weighted completion time (sum the product of the weight and completion time of each job). This can be optimally accomplished by the smallest ratio rule, which schedules the jobs in order of non-decreasing ("increasing") ratio of the amount of time required to process a job, divided by its weight. Customer relations improved even more, and Wilson was able to buy more equipment and a manufacturing building, and Wood Row has since gone multi-national. Wilson is planning to retire so he and his wife can have time to start a family. Emily is currently the vice president in charge of operations, and earning twice what her older brother makes at his successful dot-com.

Formally: In this situation, each job j has

- a processing time p_j (amount of time it will take)
- a due "date" d_j (time units may be other than days)
- a weight or priority w_j

Only one job is run at a time, starting at time=0. The time units may be considered hours. In a particular schedule (ordering of the jobs), C_j indicates the completion time of job j .

The **EDD** rule schedules jobs in order of nondecreasing d_j . This rule can be proven to minimize the maximum $C_j - d_j$.

The **ratio** rule schedules jobs in order of nondecreasing p_j/w_j . This rule can be proven to minimize $\sum_j w_j C_j$, the weighted completion time.

Analysis

In addition to the programming assignment detailed separately, you need to perform the following analysis:

Story analysis Complete the sentence: The moral of the story is _____

Algorithm analysis You can use any method to solve these (e.g. calculate by hand or by write some additional code). You must show how you arrived at your solution.

1. Using the p_j , d_j , and w_j in the code you are given, and given a week where exactly one of each item is ordered, what is the minimum maximum lateness? Which item(s) achieve this maximum lateness?
2. Same setup as previous question. How many items are always completed sooner using than ratio rule than they were using the EDD rule? What is the maximum lateness (defined with d_j used for EDD rule) when using the ratio rule? How many jobs are completed by their due date under each of the two rules?
3. When will the EDD rule give us the same results as the ratio rule? More specifically, when will the EDD rule give us an optimal solution for the weighted completion time criterion, while the ratio rule gives an optimal solution for the min max lateness rule? For example, if all jobs are the same (same p_j , d_j , and w_j), then all schedules are optimal. Can you think of any other situation? What if we set $d_j = \max_i(w_i) - w_j$ (so more weight gets translated as an earlier due date)? Are there any other modifications we can effect?