
## Outline

- Announcements:
  - Homework II: Solutions on web
  - Homework III: due Wed. by 5, by e-mail
- Structs
- Differential Equations
- Numerical solution of ODE's
- Matlab's ODE solvers
- Optimization problems
- Matlab's optimization routines

## Applications of Cell-arrays

- Strings
  - charcell={'Greetings'; 'People of Earth'}
  - disp(charcell) will output each cell on a new line
    - 'Greetings'
    - 'People of Earth'
  - Search lines using strmatch
    - strmatch('Greetings',charcell)
- Encapsulating data
  - FFTcell={a, b, dt}
    - myfft could return a cell array in this form
    - Would keep the related data together
    - But, user would have to know what is in each cell

## Structs

- Rather than storing data in elements or cells, structs store data in fields
- Better encapsulation than cell
  - FFTstrct.a=a;
  - FFTstrct.b=b;
  - FFTstrct.dt=dt;
- Can have arrays of structs
  - for j=1:n
    - strctarr(j)=StrctFunc(inputs);
  - end
  - each element must have same fields

## Working with Structs

- Lots of commands for working with structs
  - fieldnames(S)--returns a cell array of strings containing names of fields in S
  - isfield(S,'fname')--checks whether fname is a field of S
  - rmfield(S,'fname')--removes fname

## Differential Equations

- Ordinary differential equations (ODE's) arise in almost every field
- ODE's describe a function y in terms of its derivatives
- The goal is to solve for y

## Example: Logistic Growth

- N(t) is the function we want (number of animals)

$$\frac{dN}{dt} = rN(\frac{K-N}{K})$$

## Numerical Solution to ODEs

- In general, only simple (linear) ODEs can be solved analytically
- Most interesting ODEs are nonlinear, must solve numerically
- The idea is to approximate the derivatives by subtraction

## Euler Method

$$\begin{aligned} \frac{dN}{dt} &= f(N,t) \\ \frac{N_{t+1} - N_t}{\Delta t} &= f(N_t, t) \\ N_{t+1} &= N_t + \Delta t * f(N_t, t) \end{aligned}$$

## Euler Method

- Simplest ODE scheme, but not very good
- "1st order, explicit, multi-step solver"
- General multi-step solvers:

$N_{t+1} = N_t + \Delta t *$ (weighted mean of f evaluated at lots of t's)

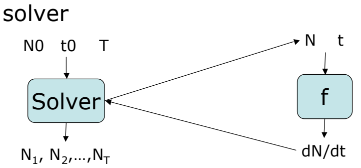
## Runge-Kutta Methods

- Multi-step solvers--each N is computed from N at several times
  - can store previous N's, so only one evaluation of f/iteration
- Runge-Kutta Methods: multiple evaluations of f/iteration:

$$
\begin{aligned}
a &= \Delta t f(N_t, t) \\
b &= \Delta t f(N_t + a/2, t + \Delta t/2) \\
c &= \Delta t f(N_t + b/2, t + \Delta t/2) \\
d &= \Delta t f(N_t + c, t + \Delta t) \\
\\
N_{t+1} &= N_t + 1/6 * (a + 2b + 2c + d)
\end{aligned}
$$

## ODE Solvers

- LMS and RK solvers are general algorithms that can work with any ODE
- Need a way to pass the function f to the solver



## Passing Functions

- One sol'n: force us to call our function "f"
- A better sol'n:
  - pass the name of the function to the solver
    - funcname="myf"
  - solver uses "feval" command
    - dN=feval(funcname,N,t);
  - This is Matlab ca. 1998

## Passing Functions

- Now, feval accepts "function handles"
  - @myf creates a handle to myf.m
- Slightly faster than strings
- Now, how do we solve the ODE's?

## Matlab's ODE solvers

- Matlab has several ODE solvers:
  - ode23 and ode45 are "standard" RK solvers
  - ode15s and ode23s are specialized for "stiff" problems
  - several others, check help ode23 or book

## Matlab's ODE solvers

- All solvers use the same syntax:
  - [t,N]=ode23(@odefile, t, N0, {options, params …})
    - odefile is the name of a function that implements f
      - function f=odefile(t, N, {params}), f is a column vector
  - t is either [start time, end time] or a vector of times where you want N
  - N0= initial conditions
  - options control how solver works (defaults or okay)
  - params= parameters to be passed to odefile

## Parameters

- To accept parameters, your ode function must be polymorphic
  - f=odefile(t,x);
  - f=odefile(t,x,params);

  - function f=odefile(t,x,params);
    - if(nargin<3)
      - params=defaults;
    - end
    - f=…

## nargin

- Matlab functions can be polymorphic--the same function can be called with different numbers and types of arguments
  - Example: plot(y), plot(x,y), plot(x,y,'rp');
- In a function, nargin and nargout are the number of inputs provided and outputs requested by the caller.
- Even more flexibility using varargin and varargout

## Example: Lorenz equations

- Simplified model of convection cells

$$\begin{aligned}\frac{dx}{dt} &= \sigma(y-x)\\ \frac{dy}{dt} &= rx - y - xz\\ \frac{dz}{dt} &= xy - bz\end{aligned}$$

- In this case, N is a vector =[x,y,z] and f must return a vector =[x',y',z']

## Optimization

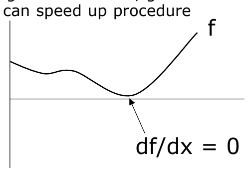
- For a function f(x), we might want to know
  - at what value of x is f smallest
  - largest?
  - equal to some value?
- All of these are optimization problems
- Matlab has several functions to perform optimization
  - Same problem as with ODE: solver needs to work with YOUR functions
  - Must pass function handle

## Optimization

- Simplest functions are
  - x=fzero(@fun, x0);%Finds f(x)==0 starting from x0
  - x=fminbnd(@fun, [xL, xR]); %Finds minimum in interval [xL,xR]
  - x=fminsearch(@fun,x0); %Finds minimum starting at x0
- All of these functions require you to write a function implementing f:
  - function f=fun(x);%computes f(x)

## Optimization Toolbox

- More optimization techniques, but used in the same way
  - Some of these functions can make use of gradient information
  - in higher dimensions, gradient tells you which way to go, can speed up procedure




## Optimization Toolbox

- To return gradient information, your function must be polymorphic:
  - x=fun(x) %return value
  - [x,dx]=fun(x); %return value and gradient
  - function [x,dx]=fun(x);
    - if(nargout>1)
      - dx=…
    - end
    - x=…

## Optimization Options

- Need to tell optimization toolbox that it can use gradient
  - Use optimset:
    - opts=optimest('ParameterName','value');
    - Then pass opts to optimization function
    - Lots of options--read docs!
  - opts=optimset('GradObj','on');%Turns gradient on
  - x=fminunc(@fun, x0,opts);%pass opts to function

## Summary

- ODE solvers & optimization routines in Matlab are “function functions”
  - You must write a function returning
    - dx/dt (ODE)
    - f (Optimization)
  - You pass the name of the function to the solver
  - Solver calls it repeatedly and returns answer