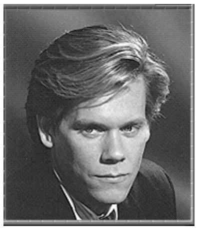
Stochastic Simulations



Six degrees of
Kevin Bacon

Outline

- Announcements:
  - Homework II: due Today. by 5, by e-mail
    - Discuss on Friday.
  - Homework III: on web
- Cookie Challenge
- Monte Carlo Simulations
- Random Numbers
- Example--Small Worlds

Homework III

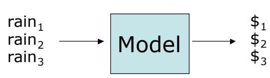
- For HW IV--you will create your own programming assignment.
  - Develop a solution (function or two) to a particular problem
  - Pick something relevant to you!
- For HW III
  - Define your problem
  - Will give me a chance to comment

Monte Carlo

- Monte Carlo methods refer to any procedure that uses random numbers
- Monte Carlo methods are inherently statistical (probabilistic)
- Used in every field
  - Galaxy formation
  - Population model
  - Economics
  - Computer algorithms

Monte Carlo Example

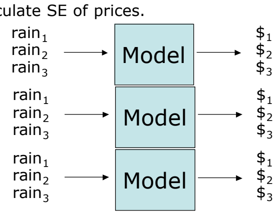
- Have a computer model which computes price of corn in Omaha using rainfall.
  - You have a forecast of rainfall for next few months from NWS
  - Forecast is rain +/- SE



- How can you incorporate uncertainty of rainfall into your forecast of prices?
  - Want $ +/- SE

Monte Carlo Example

- 1. Create several random forecast rainfall series
  - mean of the series is the forecast
  - SE of series is the forecast SE
- 2. Compute prices
- 3. Calculate SE of prices.



Random Numbers

- Computers are deterministic
  - Therefore, computers generate "pseudo-random" numbers
- Matlab's random numbers are "good"
  - "The uniform random number generator in MATLAB 5 uses a lagged Fibonacci generator, with a cache of 32 floating point numbers, combined with a shift register random integer generator."
  - http://www.mathworks.com/support/solutions/data/8542.shtml

Random functions

- rand(m,n) produces m-by-n matrix of uniformly distributed random numbers [0,1]
- randn(m,n) produces random numbers normally distributed with mean=0 and std=1
- randperm(n) is a random permutation of integers [1:n]
  - I=randperm(n); B=A(I,:) scrambles the rows of A

Seeds

- Random number generators are usually recurrence equations:
  - r(n)=F(r(n-1))
- Must provide an initial value r(0)
  - Matlab's random functions are seeded at startup, but THE SEED IS THE SAME EVERY TIME!
  - Initialize seed with rand('state', sum(100*clock))
  - How would you ensure rand is always random?

Monte Carlo Example

- 1. Create several random forecast rainfall series
  - rain, rainerr--n-by-1 vectors of rain forecasts and SE
  - P=randn(n,p)
  - randrain=P*(rainerr*ones(1,p))+(rain*ones(1,p))
- 2. Compute prices
  - for j=1:p;prices(:,j)=Model(randrain(:,j));end
- 3. Calculate SE of prices.
  - priceerr=std(prices,2)/sqrt(p);
  - pricemn=mean(prices,2);

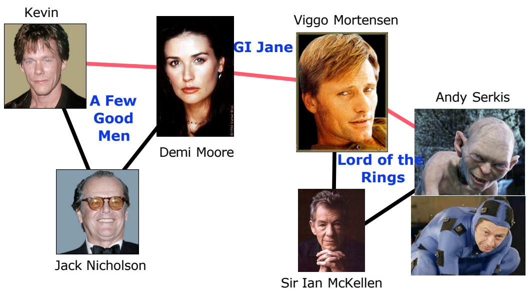
It's a Small, Small World

- Watts & Strogatz (1998) Nature, 393:440-442
- Complicated systems can be viewed as graphs
  - describe how components are connected

Example: Six Degrees of Kevin Bacon

- Components (vertices) are actors
- Connections (edges) are movies
- Hypothesis: 6 or fewer links separate Kevin Bacon from all other actors.
  - "Oracle of Bacon" at http://www.cs.virginia.edu/oracle/

Example: Kevin Bacon & Gollum



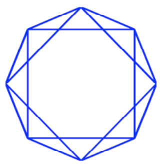
Other Systems

- Power Grid
- Food Webs
- Nervous system of *Caenorhabditis elegans*
- Goal is to learn about these systems by studying their graphs
- Many of these systems are “Small Worlds”--only a few links separate any two points

Watts & Strogatz

- Can organize graphs on a spectrum from ordered to random
- How do graph properties change across this spectrum?
  - L=mean path length (# links between points)
  - C=cluster coefficient (“lumpiness”)
- Used a Monte-Carlo approach--created lots of graphs along spectrum and computed L and C

Watts & Strogatz



- Creating the graphs
- n=# of vertices, k=number of edges/vertex
- Start with a regular ring lattice and change edges at random with probability p
- For every p, compute stats for many graphs

Small Worlds in Matlab

- G=createlattice(n,k,p)
  - creates a lattice--represented as a sparse matrix
- [L,C]=latticestats(G)
  - computes the path length and clustering stats
- [L,C]=SmallWorldsEx(n,k,P,N)
  - Creates N graphs for every P(j) and saves the mean stats in L(j) and C(j)
- plotlattice(G)
  - Plots a lattice