CS 381 First Mid Term Friday, Oct. 27, 2006
Fall 2006 Upson B17

1. Write a regular expression denoting all strings of 0's and 1's in which if there are two adjacent 0's in the string, then there will also be two adjacent 1's somewhere in the string. Give a brief English explanation as to how your regular expression works.

(0+1)*00(0+1)*11(0+1)*+
(0+1)*11(0+1)*00(0+1)*+
(ε+1)(011*)*(ε+0)

The first two terms are all strings with both 00 and 11. The third term is all strings not containing 00 as a substring.

2. State the pumping lemma for regular sets. Prove that the set L=$\left\{a^i b^j \mid i > j > 0\right\}$ is not regular using the pumping lemma for regular sets.

If R is a regular set, then there exists an integer n such that for any w in R, |w|>n, w can be written as xyz with |xy|<n, y$\neq\varepsilon$ and xy^iz is in R for all i$\geq$0.

Assume that L is a regular set. Let n be the integer of the pumping lemma. Select $w = a^{n+1}b^n$. No matter how w is written in the form xyz with |xy|<n and y$\neq\varepsilon$, y must consist solely of a's. Now xz is not in L, a contradiction. Thus the assumption that L is regular must be false.

3. Let $M = \left(Q_1, \Sigma, \delta_1, q_{01}, F_1\right)$ be a finite automaton and let $P = \left(Q_2, \Sigma, \Gamma_2, \delta_2, q_{02}, Z_{02}, F_2\right)$ be a pushdown automaton that accepts by final state. Construct a new pushdown automaton $P_3 = \left(Q_3, \Sigma, \Gamma_3, \delta_3, q_{03}, Z_{03}, F_3\right)$ such that $L\left(P_3\right) = L\left(M\right) \cap L\left(P\right)$ by specifying each of $Q_3, \Gamma_3, \delta_3, q_{03}, Z_{03},$ and F_3.

$Q_3 = Q_1 \times Q_2$

$\Gamma_3 = \Gamma_2$

$\delta_3\left([q, p], a, A\right)$ conatins $\left(\left[\text{r,s}\right], \alpha\right)$ if $\delta_1\left(\text{q,a}\right)$ contains r

and $\delta_2\left(\text{q,a,A}\right)$ contains (s,α)

$q_{03} = [q_{01}, q_{02}]$

$Z_{03} = Z_{02}$

$F_3 = F_1 \times F_2$

4. Prove that the class of context-free languages in not closed under complement. By proof we mean give a convincing argument in clear and concise English. If you use some languages which obviously are context free you do not need to prove that they are context free. If you use the fact that some language is obviously not context free, you do not need to prove that it is not context free.

The languages $a^n b^n c*$ and $a * b^n c^n$ are obviously context free. Their intersection is $a^n b^n c^n$ which obviously is not context free. If the class of context-free languages were closed under complement, then $a^n b^n c^n = \overline{\overline{a^n b^n c*} \cup \overline{a * b^n c^n}}$ would be context free, a contradiction. Therefore the class of context-free languages is not closed under complement.

5. Let L be a set of strings contained in (a+b)*. Use h, h^{-1} and intersection with a regular set to delete every copy of the substring aab in each string of L.

Let

$$h_1(a) = a \qquad h_2(a) = a$$
$$h_1(b) = b \qquad h_2(b) = b$$
$$h_1(c) = aab \qquad h_2(c) = \varepsilon$$

Then the solution is

$$h_2\left(h_1^{-1}(L) \cap \left((a+b+c)^* - (a+b)^* aab (a+b)^*\right)\right)$$